ELEMENTS

OF

MACHINE DESIGN,

NOTES AND PLATES,

FOR THE USE OF STUDENTS IN

LEHIGH UNIVERSITY,

BY

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THIRD EDITION.

BETHLEHEM, PA.,
KEYSTONE PRINTING CO.
1901.

PREFACE.

THESE Notes and Plates constitute a short course in designing the most important of the Machine Parts that occur in practice. The proportions are either inscribed on the plates or are given by the formulas and tables; the dimensions are to be calculated in all cases from the data given in the text. The problems have been so chosen, that each plate can be easily drawn on a 26″x40″ sheet, but for most students it will be found advantageous to have each machine piece drawn full size.

As this is not a complete treatise, even on the Machine Parts represented, frequent reference to the larger works of *Reuleaux* and *Unwin* will be found desirable.

Numerous tables expedite the work of calculation and special attention is called to the new and exact tables for finding speed cone diameters and for laying out exactly the profiles of teeth by means of rectangular co-ordinates.

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NOTES ON MACHINE DESIGN

FOR THE

USE OF STUDENTS

IN

LEHIGH UNIVERSITY.

TABLE FOR CONVERTING DECIMALS INTO FRACTIONS.

4th	8th	16th	32d	64th		4th	8th	16th	32d	64th	
				1	.015625					33	.515625
			1		.031250				17		.531250
				3	.046875					35	.546875
		1			.062500			9			.562500
				5	.078125					37	.578125
			3		.093750				19		.593750
				7	.109375					39	.609375
	1				.125000		5				.625000
				9	.140625					41	.640625
			5		.156250				21		.656250
				11	.171875					43	.671875
		3			.187500			11			.687500
				13	.203125					45	.703125
			7		.218750				23		.718750
				15	.234375					47	.734375
1					.250000	3					.750000
				17	.265625					49	.765625
			9		.281250				25		.781250
				19	.296875					51	.796875
		5			.312500			13			.812500
				21	.328125					53	.828125
			11		.343750				27		.843750
				23	.359375					55	.859315
	3				.375000		7				.875000
				25	.390625					57	.890625
			13		.406250				29		.906250
				27	.421875					59	.921875
		7			.437500			15			.937500
				29	.453125					61	.953125
			15		.468750				31		.968750
				31	.484375					63	.984375
2					.500000	4					1.000000

PLATE I.

FASTENINGS.

d = diameter of bolt.

d_1 = diameter of core = diameter across the bottom of threads.

D = actual external diameter (of the gas or water pipe) throughout its parallel length, expressed in inches.

N = least number of threads that will prevent rapid wear in screws used for communicating motion.

n = number of threads per inch in screws used for fastenings.

P = effective working load in pounds.

p = pitch of thread.

t = actual depth of thread.

t_0 = depth of thread when not truncated at top and bottom. (see Plate I).

In screws used for fastenings,

$$d_1 = 0.0183\sqrt{P}, \qquad (1)$$

where P = working load;

$$d = d_1 + 2t. \qquad (2)$$

The working stress per □″ in this formula is taken equal to 4000 pounds, and the bolts are assumed to be fitted accurately and tightened moderately.

When the joints are to be steam tight,

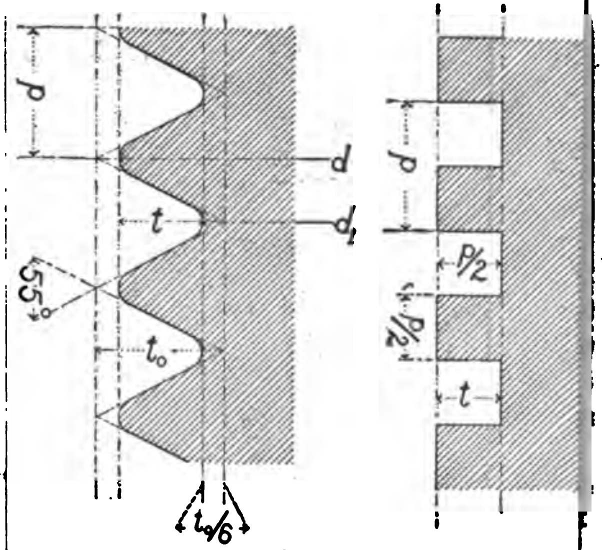
$$d_1 = 0.025\sqrt{P} \qquad (3)$$

the allowable working stress per □″ being 2000 pounds.

The diameter of the screw or bolt is rarely determined directly from the tension in the direction of its length. Usually the diameter of the bolt is referred to certain parts of the pieces to be fastened, and then the principal dimension of these parts is

ENGLISH.

WHITWORTH THREAD. SQUARE.



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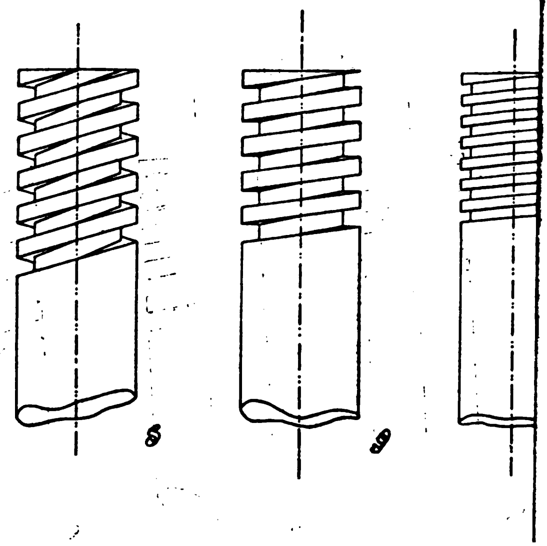
FASTENIN

SECTION OF SCR

CONVENTIONAL METHOD OF RE

DOUBLE. SINGLE.



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chosen as the unit of reference—for example, the thickness of plates which are to be fastened together is taken as the unit of reference. See Unwin, pp. 148–155; Reuleaux, p. 235, German Ed., and pp. 243–247, French Ed.

The srews used for transmitting motion have square or trapezoidal threads; for both of them,

$$d_1 = .012\sqrt{P} \text{ is minimum value,} \tag{4}$$

$$d_1 = .0183\sqrt{P} \text{ is usual value,} \tag{5}$$

$$t = \frac{d}{10} = \frac{d_1}{8} \text{ is usual value.} \tag{6}$$

If N is *least* number of threads in nut that is necessary to prevent rapid wear, then will

$$N = 0.00045\frac{P}{d^2}\,\frac{1}{\frac{t}{d}\left(1 - \frac{t}{d}\right)}; \tag{7}$$

this corresponds to a pressure of 700 pounds per □″ of projected bearing surface of threads, the plane of projection being at right angles to axis of screw; substituting

$$t = \frac{d}{10} = \frac{d_1}{8} \text{ we get } N = \frac{P}{300d_1^2} = 0.0051\frac{P}{d^2}, \tag{8}$$

whatever the formula may give we must not take N less than 12.

Fig. 1.—*American or Sellers Thread.*

$$p = 0.24\sqrt{d + 0.625} - 0.175 \tag{9}$$

$$n = \frac{1}{p} = \text{number of threads per inch.} \tag{10}$$

$$d_1 = d - 0.312\sqrt{d + 0.625} + 0.227 \tag{11}$$

$$= d - \frac{1.299}{n} \tag{12}$$

$$t_0 = 0.866p \tag{13}$$

$$t = \tfrac{3}{4}t_0 = 0.65p. \tag{14}$$

Fig. 2.—*English or Whitworth Thread.*

$$p = 0.08d + 0.04 \qquad (15)$$

$$d_1 = 0.9d - 0.05 \qquad (16)$$

$$t_0 = 0.96p \qquad (17)$$

$$t = \tfrac{2}{3}t_0 = 0.64p. \qquad (18)$$

Fig. 3.—*Square Thread.*

When used for fastenings,

$$p = 0.08 + 0.09d \qquad (19)$$

$$t = \frac{p}{2} \qquad (20)$$

$$d_1 = 0.91d - 0.08. \qquad (21)$$

For communicating motion,

$$p = \frac{d}{5} = \frac{d_1}{4} \qquad (22)$$

$$t = \frac{d}{10} = \frac{d_1}{8} = \frac{p}{2} \qquad (23)$$

Fig. 4.—*Trapezoidal Thread.*

When used for fastenings,

$$p = 0.053 + 0.06d \qquad (24)$$

$$t = \tfrac{3}{4}p \qquad (25)$$

$$d_1 = 0.91d - 0.08 \qquad (26)$$

For communicating motion,

$$p = \frac{2d}{15} = \frac{d_1}{6} \qquad (27)$$

$$t = \frac{d}{10} = \frac{d_1}{8} \qquad (28)$$

A slight rise is given to thread, $4\frac{1}{2}°$ for $d = \frac{1}{4}$ inches, $1\frac{1}{2}°$ for $d = 3$ in., so that the pressure along the axis will have no tendency to make the screw move backward.

In many cases the diameter of a screw is taken greater than that of the normal screw or bolt; these enlarged screws have the same thread section as the normal ones and are used in special cases for stuffing boxes and pipe connections.

STANDARD DIMENSIONS OF STEAM, GAS AND WATER PIPES.

Nominal Diameter.	Thickness.	Actual inside Diameter.	No. of Threads per inch of Screw n	Pitch of screw $p=\frac{1}{n}$.
$\frac{1}{8}$	.068	.270	27	.037
$\frac{1}{4}$	.088	.364	18	.056
$\frac{3}{8}$	.091	.494	18	.056
$\frac{1}{2}$	.109	.623	14	.071
$\frac{3}{4}$	.113	.824	14	.071
1	.134	1.048	$11\frac{1}{2}$	.087
$1\frac{1}{4}$	.140	1.380	$11\frac{1}{2}$	.087
$1\frac{1}{2}$	.145	1.611	$11\frac{1}{2}$	.087
2	.154	2.067	$11\frac{1}{2}$	.087
$2\frac{1}{2}$	.204	2.468	8	.125
3	.217	3.061	8	.125
$3\frac{1}{2}$	.226	3.548	8	.125
4	.237	4.026	8	.125
$4\frac{1}{2}$	.247	4.508	8	.125
5	.259	5.045	8	.125
6	.280	6.065	8	.125
7	.301	7.023	8	.125
8	.322	7.982	8	.125
9	.344	9.001	8	.125
10	.366	10.019	8	.125

Taper of conical tube ends is 1 in 32 to axis of tube. Length of tapered portion is equal to $\frac{0.8D + 4.8}{n}$ = also portion throughout which the screw-thread continues perfect. See Report of Committee on Standard Pipe and Pipe Threads, Trans. Am. Soc. Mechanical Engineers, Vol. VIII.

Fig. 5.—*Thread for $2\frac{1}{2}''$ Pipe.*

PLATE II.

FASTENINGS.

D = width of nut across the flats = distance between two parallel sides.

D_1 = width of nut across corners = diagonal of hexagon.

D_2 = diameter of washer.

d = diameter of bolt.

h = thickness (or height) of bolt head.

h_1 = thickness (or height) of nut.

t = thickness of washer.

The diameter of the bolts may be determined from the formula given at the beginning of the notes for Plate I. When the diameter of the bolt is not determined from the tensile stress to which it is subjected, it must be determined with reference to the thickness of the pieces which it connects. When it is possible bolts should be strained in the direction of their length only. Special precautions should be taken to prevent lateral or shearing stresses. The heads and nuts of bolts should generally rest on finished surfaces. Cast pieces may be provided with faces (ledges or bosses) which alone need to be planed or turned when the piece is being fitted. See Reuleaux' "Konstructeur," pp. 233 – 235, German Ed. and pp. 243 – 247, French Ed. Also Unwin, p. 164.

Fig. 1.—*Bolt with Hexagonal Nut and Head.*

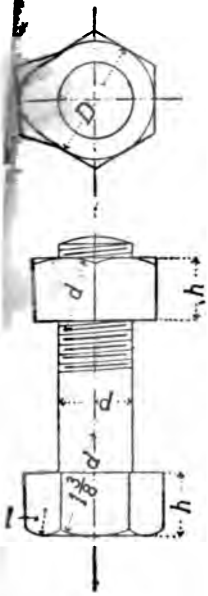
For finished work,

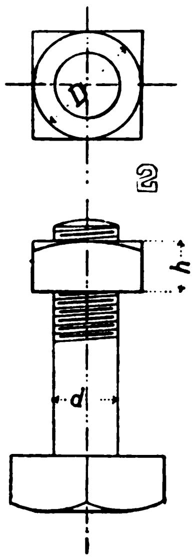
$$D = 1\tfrac{1}{2}d + \tfrac{1}{16}, \tag{29}$$

$D = 2d$ may be used for small work. (30)

$$h = d - \tfrac{1}{16}, \tag{31}$$

$$h_1 = h = d - \tfrac{1}{16}, \tag{32}$$





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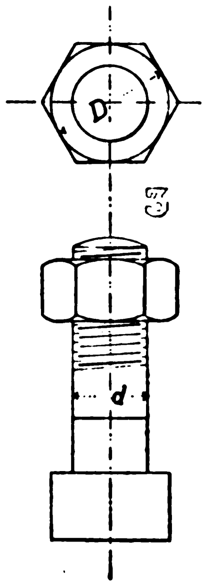

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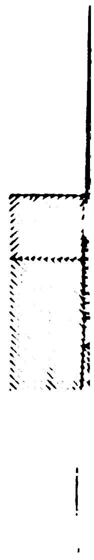

FASTENINGS.

BOLTS AND NUTS.

METHODS OF SECURING NUTS.

II

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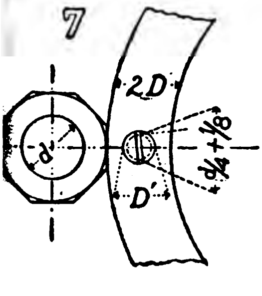

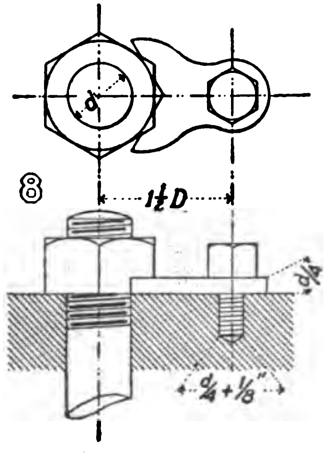

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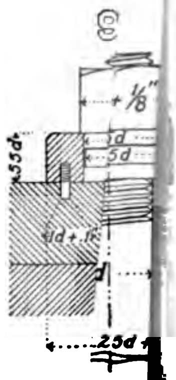


For rough work,

$$D = 1\tfrac{1}{2}d + \tfrac{1}{8}, \tag{33}$$
$$h = \tfrac{1}{2}D, \tag{34}$$
$$h_1 = d. \tag{35}$$

The corners of one end of both head and nut (sometimes in rough work for both ends of nut) are chamfered by a cone, the diameter of whose base $= D$, and angle at the base $= 30°$, this base lying in the end plane of the nut or head. The corners are sometimes chamfered by a sphere of radius $\tfrac{5}{8}D$, as in Reuleaux' "Konstructeur," p. 211, Fig. 218, German Ed. and p. 219, Fig. 216, French Ed.

Nuts are generally made hexagonal, for less careful work they are sometimes square. Height of nut and distance between flats are empirical and are referred to diameter of bolt as above. Numerous experiments in England* have shown that on an average the height of nut must be one-third of diameter of bolt, in order that the threads may have the same resistance to shearing that the inner core of the screw has to tearing. The same result is obtained by calculation. In practice, however, the nuts are made higher then strength requires in order that a thread may be cut which will securely guide the nut on the bolt, reduce the pressure on the threads and enable a wrench to be easily applied. When nuts are often unscrewed the wear on the threads is diminished by making,

$$h = \frac{4}{3}d.$$

Draw one of the 10 bolts necessary to fasten a cylinder cover to the head of a steam cylinder; diameter of cylinder $= 12''$, pressure of steam $= 70$ pounds per $\square''$ (gauge pressure).

Fig. 2.—*Bolt with Square Head and Nut.*

The same formula to be used as for bolt with hexagonal head and nut. Draw $\tfrac{3}{4}''$ bolt, rough.

* See also Stevens Institute experiments; Railroad Gazette, 1877, p. 483.

Fig. 3.—*Bolt with Round Head, Square Shank, and Hexagonal Nut chamfered on both ends.*

In this bolt, part of the shank and the hole into which it fits, are made square in order that the bolt may not turn while the nut is being screwed down. Various devices for accomplishing the same purpose may be seen in Reuleaux, pp. 216–217, German Ed., and pp. 224-226, French Ed., also Unwin, p. 159.

Fig. 4.—*Stud Bolt with Left-handed Thread.*

A stud bolt has no head. It is screwed into one of the two connected pieces and remains in position after the nut is removed. It is screwed into position by means of two nuts screwed against each other on the end of the bolt. These nuts act on each other as check nuts, each preventing the other from rotating relatively to the shank. A wrench is applied to the upper one of the two nuts and the stud bolt screwed into position. Stud bolts may have either right or left-handed threads. A thread is right-handed when (the screw being held vertically before the eyes) on the side of the screw nearest the observer the thread slants in the same direction as ordinary writing. A thread is left-handed when it slants in the opposite direction.

There are still other kinds of nuts used for special purposes, see Unwin, p. 158. Bolts with variously shaped heads are represented and explained by Reuleaux, p. 216. We will call special attention only to the foundation bolts suitable for steam engines in Reuleaux, p. 217, Fig. 227, German Ed., and p. 225, Fig. 225, French Ed.

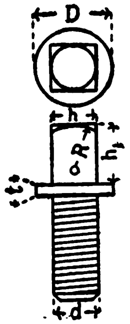
Set screws are bolts which do not require any nuts, they are used either for holding pieces or for moving them.

Slotted-head screws are to be fastened down as an ordinary wood screw, by means of a screw-driver.

The following table represents the form of set screws and slotted-head screws used in Colt's Armory, Hartford.

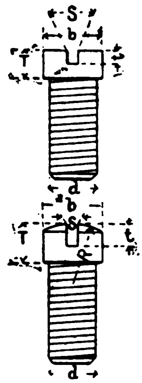
Set Screws.

$R=h=d$	D	h_1	t	No. of Threads.
$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{16}$	22
$\frac{5}{16}$	$\frac{9}{16}$	$\frac{7}{16}$	$\frac{3}{32}$	20
$\frac{3}{8}$	$\frac{19}{32}$	$\frac{1}{2}$	$\frac{7}{64}$	18
$\frac{7}{16}$	$\frac{21}{32}$	$\frac{1}{2}$	$\frac{1}{8}$	16
$\frac{1}{2}$	$\frac{13}{16}$	$\frac{5}{8}$	$\frac{1}{8}$	14
$\frac{9}{16}$	$\frac{15}{16}$	$\frac{3}{4}$	$\frac{5}{32}$	12
$\frac{5}{8}$	$\frac{15}{16}$	$\frac{3}{4}$	$\frac{5}{32}$	12
$\frac{3}{4}$	$1\frac{1}{8}$	$\frac{7}{8}$	$\frac{1}{4}$	12



Slotted-head Screws.

d	$R=b$	T	t	S	No. of Threads.
$\frac{1}{4}$	$\frac{11}{32}$	$\frac{1}{4}$	$\frac{1}{16}$	$\frac{3}{64}$	22
$\frac{5}{16}$	$\frac{13}{32}$	$\frac{1}{4}$	$\frac{3}{32}$	$\frac{3}{64}$	20
$\frac{3}{8}$	$\frac{15}{32}$	$\frac{1}{4}$	$\frac{7}{64}$	$\frac{1}{16}$	18
$\frac{7}{16}$	$\frac{19}{32}$	$\frac{5}{16}$	$\frac{7}{64}$	$\frac{5}{64}$	16
$\frac{1}{2}$	$\frac{21}{32}$	$\frac{5}{16}$	$\frac{5}{32}$	$\frac{3}{32}$	14
$\frac{9}{16}$	$\frac{23}{32}$	$\frac{3}{8}$	$\frac{5}{32}$	$\frac{3}{32}$	12
$\frac{5}{8}$	$\frac{27}{32}$	$\frac{7}{16}$	$\frac{11}{64}$	$\frac{7}{64}$	12
$\frac{11}{16}$	$\frac{31}{32}$	$\frac{1}{2}$	$\frac{3}{16}$	$\frac{9}{64}$	12
$\frac{3}{4}$	$1\frac{7}{32}$	$\frac{5}{8}$	$\frac{1}{4}$	$\frac{3}{16}$	12
$\frac{3}{4}$	$1\frac{1}{4}$	$\frac{5}{8}$	$\frac{1}{4}$	$\frac{3}{16}$	12



The Securing of Nuts.

Nuts even when well screwed down are liable to get loose whenever the machine part to which they are attached is supjected to shocks or vibrations. Various devices have been employed to prevent the nuts from getting loose. The arrangements shown in Figs. 5, 6, 7, 8 and 9 are all for this purpose. For other arrangements for locking nuts see Unwin, pp. 161–163, and Reuleaux, pp. 220–224, German Ed., and 228–236, French Ed.

Fig. 5.—*Check Nut.*

By first screwing down the check nut (or smaller nut against the plate to be fastened down, and then the larger nut) the stress-

will be better distributed, the greater stress coming on the outside nut. Often the larger one is screwed down first as a matter of convenience in using the wrench. The friction of two nuts is greater than that of a single nut and its bolt. The upper nut can be screwed more tightly against the lower than it is desirable to screw the latter against its connecting pieces. This allows the friction to be greater than with single nut. Weisbach-Herrmann, Eng. Edition, Vol. III, Part I, Section II, p. 607. Though this device is much used it does not establish an absolute safeguard against loosening of nut. Assume diameter of bolt $= \frac{3}{4}$ inches.

Fig. 6.—*Nut Secured by Split Pin.*

A hole is drilled through the bolt above, and close to the nut, the pin drawn through and its ends turned outward, as in the figure. Split pins are generally made of half round iron. Assume $d = \frac{3}{4}$ inches.

Fig. 7.—*Nut Secured by Flat Plate and Set Screw.*

One flat plate, properly made, may be used to secure a number of nuts. The annular flat plate represented by the figure would be suitable for securing nuts set in a circle.

Fig. 8.—*Nuts Secured by Stop Plate Fixed on one Side.*

By unscrewing the stop plate the nut may be tightened at least by one-twelfth of a turn. Assume $d = \frac{7}{8}$ inches.

Fig. 9.—*Nuts Secured by Stop Ring and Set Screw.*

This allows of any amount of tightening, and would be useful in pedestals for taking up the wear of the brasses. Assume $d = 1\frac{1}{2}$ inches.

Fig. 10.—*Diagram for Determining the Dimensions of Bolts and Nuts.*

In this diagram the diameters of bolts are the ordinates, and the required dimensions the abscissas. Each dimension may be

obtained from the diagonal line which represents its relation to the diameter of the bolt. The diagonals are all straight lines, because the equations expressing the relation between the dimensions and the diameter of the bolt, are all of the first degree.

This diagram and similar ones are best and most easily made on section paper, accompanied by a figure showing all the dimensions which it determines.

The figure accompanying the diagram has shown on it a washer or collar whose function it is to furnish a good support for the nuts, equalize the stress and cover elongated holes.

$$t = \frac{D}{10}, \tag{36}$$

$$D_2 = \frac{4}{3}D. \tag{37}$$

Nuts are usually made by special machines which turn them out in a condition to be tapped.

Bolts are also made by special machines so that on leaving the latter they are ready to be turned and have thread cut by means of cutting machines. Or they may be forged by hand from a round bar of wrought iron, one end of which is upset to form the head, which last is further transformed by suitable dies. It may then be turned and have its thread cut on an ordinary lathe. For a good discussion of the stresses and strains to which a screw (used for fastening or for producing motion) is subjected, see Weisbach-Herrmann, English Edition, Vol. III, Part I, Section II, pp. 594–604.

Weight of Bolts, Shank 1″ long.			
Diam. of Bolt.	Weight of Shank 1″ long.	Diam. of Bolt.	Weight of Shank 1″ long.
	lbs.		lbs.
$\frac{1}{8}$	0.003455	$\frac{11}{16}$	0.104530
$\frac{3}{16}$	0.007776	$\frac{3}{4}$	0.124383
$\frac{1}{4}$	0.013821	$\frac{13}{16}$	0.145981
$\frac{5}{16}$	0.021596	$\frac{7}{8}$	0.169326
$\frac{3}{8}$	0.031089	1	0.221169
$\frac{7}{16}$	0.042324	$1\frac{1}{8}$	0.2799
$\frac{1}{2}$	0.045278	$1\frac{1}{4}$	0.3455
$\frac{9}{16}$	0.069978	$1\frac{3}{8}$	0.4179
$\frac{5}{8}$	0.086367	$1\frac{1}{2}$	0.4976

SQUARE AND HEXAGON NUTS.

Chamfered and Trimmed.

Number of each Size in 100 Pounds.

Size of Bolt.	Thickness.	Hole.	Width.	No. of Square.	No. of Hexagon.
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{13}{64}$	$\frac{1}{2}$	8140	9300
$\frac{5}{16}$	$\frac{5}{16}$	$\frac{1}{4}$	$\frac{19}{32}$	3000	6200
$\frac{3}{8}$	$\frac{3}{8}$	$\frac{5}{16}$	$\frac{11}{16}$	2320	3100
$\frac{7}{16}$	$\frac{7}{16}$	$\frac{11}{32}$	$\frac{23}{32}$	1740	2200
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{13}{32}$	$\frac{7}{8}$	1180	1350
$\frac{9}{16}$	$\frac{9}{16}$	$\frac{29}{64}$	$\frac{31}{32}$	920	1000
$\frac{5}{8}$	$\frac{5}{8}$	$\frac{33}{64}$	$1\frac{1}{16}$	728	830
$\frac{3}{4}$	$\frac{3}{4}$	$\frac{5}{8}$	$1\frac{1}{4}$	420	488
$\frac{7}{8}$	$\frac{7}{8}$	$\frac{23}{32}$	$1\frac{7}{16}$	280	309
1	1	$\frac{27}{32}$	$1\frac{5}{8}$	180	216
$1\frac{1}{8}$	$1\frac{1}{8}$	$\frac{15}{16}$	$1\frac{13}{16}$	130	148
$1\frac{1}{4}$	$1\frac{1}{4}$	$1\frac{1}{16}$	2	96	111
$1\frac{3}{8}$	$1\frac{3}{8}$	$1\frac{5}{32}$	$2\frac{3}{16}$	70	85
$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{9}{32}$	$2\frac{3}{8}$	50	70

BOLTS WITH SQUARE HEADS AND NUTS.

Weight of 100 of Enumerated Sizes.

Length.	$\frac{1}{4}$ in.	$\frac{3}{8}$ in.	$\frac{1}{2}$ in.	$\frac{5}{8}$ in.	$\frac{3}{4}$ in.	$\frac{7}{8}$ in.	1 in.	$1\frac{1}{8}$ in.
$1\frac{1}{2}$	4.16	10.62	23.87	39.31				
$1\frac{3}{4}$	4.22	11.72	25.06	41.38				
2	4.75	12.38	26.44	45.69	73.62			
$2\frac{1}{4}$	5.34	12.90	28.62	49.50	76.			
$2\frac{1}{2}$	5.97	14.69	29.50	51.25	79.75			
$2\frac{3}{4}$	6.50	16.47	31.16	53.	83.			
3		17.87	32.44	56.	85.38	127.25		
$3\frac{1}{2}$		18.94	39.75	63.12	93.44	140.56		
4		20.59	42.50	74.87	108.12	148.37	228	296
$4\frac{1}{2}$		21.69	44.87	79.62	113.12	158.76	239	310
5		23.62	48.81	83.	118.	167.25	250	324
$5\frac{1}{2}$		25.81	51.38	87.38	123.62	174.88	261	338
6		26.87	53.31	92.38	131.75	204.25	272	352
$6\frac{1}{2}$			56.87	96.88	139.56	214.69	283	366
7			59.12	99.87	145.50	228.44	294	370
$7\frac{1}{2}$			61.87	105.75	150.88	235.31	305	384
8			64.44	109.50	157.12	239.88	316	398
9			70.50	118.12	169.62	258.12	338	426
10			77.00	128.13	184.	276.18	360	454
11			82.88	136.19	195.13	295.69	382	482
12			86.37	144.87	209.75	311.94	404	510

PLATE III.

FASTENINGS.

[illegible] = largest diameter in head of rivet.
[illegible] = largest diameter in head of rivet when countersunk.
[illegible] = largest diameter in tail of rivet.
[illegible] = smallest diameter in tail of rivet
[illegible] = diameter of rivet.
D = diameter of shell of boiler in feet.
D_1 = diameter of flue tube in inches.
E = efficiency of joint in single riveting.
E' = efficiency of joint in double riveting.
f = permissible working stress on joint per □″ of entire plate
h = height of rivet head.
h_1 = height of tail of rivet.
l = length of rivet shank necessary to fill hole and form head
L = distance apart of the stiffening rings in inches.
L_1 = total length of tube in inches.
n = number of lengths into which tube is divided by stiffening rings.
p = pitch of rivets in single riveting.
p^1 = pitch of rivets in double riveting, measured on line of rivets.
p_1 = perpendicular distance between two lines of riveting in zigzag riveting.
P = permissible working pressure in boiler shell in pounds per □″.
P_1 = external (collapsing) pressure in pounds per □″ when flue tube has no stiffening rings.
P_2 = bursting pressure of boiler shell in pounds per □″.

s = the lap of the plate, measured from center line of rivets to edge of plate.
t = thickness of plate.
t_1 = thickness of plate on flue tube.
T = total thickness of plates at joint.

Rivets and Riveted Joints for Wrought Iron Boiler Plates.

Fairbairn found by experiment that the strength of a wrought iron boiler (shell) with single riveted joint was 34000 pounds per □″ of the entire plate.

A close approximation to the theoretical formula for the thickness of boiler plate when subjected to internal pressure is as follows:

$$t = \frac{6PD}{f} \tag{38}$$

This formula makes no allowance for corrosion.

Rankine takes the working pressure P at one-sixth of the ultimate resistance, which Fairbairn finds to be 34000 pounds thus making $f = 5670$ pounds; hence we have

$$t = 0.00106PD, \tag{39}$$

which also expresses rule of the United States Inspectors for determining the thickness of boiler plates.

Prof. Radinger gives for the thickness of the European boilers at the Vienna Exposition,

$$t = 0.000912PD + \tfrac{1}{8} \text{ inches.} \tag{40}$$

If we assume the strength of joint equal to 34000 pounds per □″ of the entire plate, then of the two formulas

$$t = \frac{PD}{1000} + \tfrac{1}{8}, \tag{41}$$

$$t = \frac{\tfrac{3}{4}PD}{1000} + \tfrac{1}{8}, \tag{42}$$

the first corresponds to a factor of safety of 6 with an additional $\frac{1}{8}$ inch for corrosion, while the second corresponds to a factor of safety of $4\frac{1}{2}$ with the same additional allowance for corrosion.

When a tube or flue is exposed to an external or collapsing pressure, none of the above formulas can be employed to determine the thickness of plate. An empirical formula deduced from Fairbairn's experiments must be used instead. Both these experiments and theoretical investigation show that the *length* of the tube has an important influence on its strength, the strength increasing as the length decreases. The two empirical formulas which have been deduced from Fairbairn's experiments are as follows:

$$P_1 = \frac{96720000 t_1^{2.19}}{L_1 D_1} \tag{43}$$

also

$$P_1 = 15547000 \frac{t_1^{2.35}}{L_1^{0.9} D_1^{1.16}} \tag{44}$$

and

$$\frac{5358150}{L_1 D_1} t_1^2 + \frac{41906}{D_1} t_1^2 - 1323 \frac{t_1}{D_1} \tag{45}$$

In order that the boiler and the flue may be of the same strength, we should have

$$P_2 = \frac{5700t}{D} \tag{46}$$

$$\frac{P_2}{P_1} = n \tag{47}$$

$$L = \frac{L_1}{n} \tag{48}$$

The thickness of the small fire-tubes used in locomotive boilers is sufficient to prevent their burning out or corroding rapidly; they are then much more than strong enough to resist the collapsing pressure. See Reuleaux, p. 707, Figs. 713–714 of French Edition, and p. 1084, Fig. 1119, 4th German Edition, (1882–1889).

GENERAL FORMULAS FOR RIVETS

when $t \geqq \frac{1}{8}$ inch and $t \overline{\overline{<}} \frac{3}{4}$ inch.

$$d = \frac{5}{16} + t \qquad (49)$$

$$l = 1\tfrac{1}{2}d \text{ (or from } 1.25d \text{ to } 1.7d) \qquad (50)$$

$$h = \tfrac{5}{8}d \text{ (or from } \tfrac{1}{2}d \text{ to } \tfrac{3}{4}d) \qquad (51)$$

$$h_1 = h = \tfrac{5}{8}d \qquad (52)$$

$$h_2 = \tfrac{1}{2}d \qquad (53)$$

$$b = 2d \qquad (54)$$

$$b' = 1\tfrac{1}{2}d \qquad (55)$$

$$b_1 = 1.7d \qquad (56)$$

$$b_2 = 1.2d \qquad (57)$$

When the rivet is to connect a number of plates,

$$d = \frac{T}{8} + \tfrac{5}{8}, \qquad (58)$$

T representing the total thickness of plates.

Fig. 1.—*Cone Head Rivet.*

This is an ordinary rivet with conical head formed by hand. This head is often made higher (h greater) where it is exposed to view. A table of *Weights* of these rivets will be given on page 23.

Fig. 2.—*Snap Head Rivet.*

The head of this rivet is either machine made or it is formed by means of a cup-shaped die driven against the work by a sledge hammer. In this figure the edges of the hole are rounded, considerably increasing the strength of the joint by preventing a cutting action on the rivet.

Fig. 3.—*Counter-Sunk Rivet.*

This rivet is used when the surface of the plate must be without projections.

Fig. 4.—*Rivet for Properly Punched Plate.*

The metal surrounding the holes of plates is usually injured (as ordinarily punched) by the severe compression to which it is subjected during the operation. Experiment shows that the diameter of the bolster is one-fourth larger than the punch.

FORMULAS FOR SINGLE RIVETED JOINTS

when $t > \frac{1}{8}$ and $t \overline{\overline{<}} \frac{3}{4}$ inch.

$$s = \tfrac{7}{16} + d = t + \tfrac{3}{4} > 1\tfrac{1}{2}d, \qquad (59)$$

$$p = \left(1 + \tfrac{7}{8}\frac{d}{t}\right)d. \qquad (60)$$

FORMULAS FOR DOUBLE RIVETED JOINTS

when $t > \frac{1}{8}$ inch and $t \overline{\overline{<}} \frac{3}{4}$ inch.

$$s = d + \tfrac{7}{16} = t + \tfrac{3}{4} > 1\tfrac{1}{2}d, \qquad (61)$$

$$p^1 = \left(1 + \tfrac{7}{4}\frac{d}{t}\right)d, \qquad (62)$$

$$p_1 = 1.6d \text{ when } t \overline{\overline{<}} \tfrac{1}{2}. \qquad (63)$$

For very complete treatment of riveted joints see Unwin, Chapter IV.

LAP AND BUTT RIVETING.

"When one plate is made to overlap the other and one or more lines of rivets are put through the two, the riveting is *lap* riveting. When the plates are kept in the same plane and a cover plate is put over the joint and riveted to each, the riveting is called *butt* riveting."

SINGLE AND DOUBLE RIVETING.

"If there is one line of rivets in lap riveting, or one on each side of the joint in butt riveting, the joint is single riveted. If there are two lines of rivets in lap riveting, or two lines on each side of the joint in butt riveting, the joint is double riveted."

Fig. 5.—*Single Riveted Lap Joint.*

The over-lap s and pitch of rivet p can be obtained from the preceding formulas. This form of joints is much used in boilers It has one objection, in that it tends, indirectly, to groove the boiler plate by bending it. The bending force is a couple and is due to the fact that the tensile forces acting along the plates are not directly opposed.

Fig. 6.—*Single Riveted Butt Joint.*

Grooving of boiler plate may also occur with this form of joint, the forces tending to bend the covering plate.

Fig. 7.—*Double Riveted Lap Joint.*

By having a double line of rivets the net section on each line is increased and consequently the efficiency $\frac{p' - d}{p'}$ of the joint is also increased. This form of double riveting is also called zigzag or staggered riveting.

Fig. 8.—*Three-plate Joint, Single Riveted.*

No one seam extends eompletely around the boiler, each terminates in the longitudinal seam; in other words, the cross seams "break joint." Where a cross seam meets a longitudinal seam three plates must be riveted together. In order that this may be neatly done the edge of middle plate must be thinned out, as shown in section. The edge to be thinned is first heated red-hot and then hammered to the required thickness.

Fig. 9.—*Three-plate Lap Joint with Single Riveted Cross-Seam and Double Riveted Longitudinal Seam.*

The longitudinal seam is generally double riveted, because it is subjected to much greater stress than the cross-seams.

Fig. 10.—*Three-plate Butt Joint, Single Riveted.*

By putting one cover plate on the outside and the other on the inside, the seams can be well calked and a staunch joint obtained.

The choice of diameter of the rivet depends upon

(*a*) the bearing resistance of the rivet hole (*i. e.* the resistance to crushing at the surface of the hole);

(*b*) the shearing resistance of plate to punching, and

(*c*) the resistance to crushing of steel punch.

If we assume that the resistance to shearing of rivet should equal the bearing resistance of hole, we get,

$$d = 2.55t, \tag{64}$$

when the shearing strength of rivet per $\square''$ is equal to tensile strength of plate.

But when shearing strength of the rivet is $\frac{4}{5}$ of the tensile strength of the plate, we get,

$$d = 3.2t. \tag{65}$$

When d is taken smaller than any of these values the bearing resistance will be in excess. If we assume that the resistance to crushing of steel punch is equal to four times the resistance of the plate to shearing (the ratio of the two stresses is probably greater when punch is of good steel), we shall have,

$$d > t. \tag{66}$$

European engineers usually assume,

$$d = 2t. \tag{67}$$

The value chosen above,

$$d = t + \tfrac{5}{16}. \tag{68}$$

corresponds well with American practice for the limits $t > \frac{1}{8}$ inches and $t =$ or $< \frac{3}{4}$ inches, it gives moreover a suitable diameter for the punch and removes all danger of burring or elongating the rivet holes.

For the thicker plates, d might with advantage be made larger (particularly if the joints are to be machine riveted), the larger rivets and pitches corresponding to greater efficiency of joint.

We may assume that the cross section of the rivets is equal to, rather than less than, the net section of the plate, for the following reasons:

(*a*) In plates the shearing in the practically important direction is often found equal to, and even greater than, the tensile strength in the direction of the fiber;

(*b*) Rivet iron is generally better in quality than ordinary rolled iron;

(*c*) Loss of strength undergone by the plate in perforation;

(*d*) The rivet is less exposed to corrosion than the plate;

(*e*) Some dependence may be placed on the friction between the plates called forth by the grip of the rivets;

These reasons justify us in assuming,

cross section of rivets = net section of plate.

We then get for single riveting,

$$p = \left(1 + 0.785\frac{d}{t}\right)d, \tag{69}$$

and for double riveting,

$$p' = \left(1 + 1.57\frac{d}{t}\right)d. \tag{70}$$

When the stress upon the plate is perpendicular to the direction of the fiber and the tensile strength is $\frac{9}{10}$ of that in the direction of the fiber we get,

for single riveting, $p = \left(1 + \frac{7}{8}\frac{d}{t}\right)d,$ (71)

for double riveting, $p' = \left(1 + \frac{7}{4}\frac{d}{t}\right)d.$ (72)

The last two formulas were given above for calculating the pitches occurring in the various figures of this plate.

The ratio of the effective cross section is also called its efficiency and is represented by,

$$E = \frac{p - d}{p}, \tag{73}$$

$$E' = \frac{p' - d}{p'}, \tag{74}$$

When $\frac{d}{t} = 1.5 \quad 2 \quad 2.5 \quad 3,$

$E = 0.57 \ 0.64 \ 0.69 \ 0.72$, for single riveting, (75)

$E' = 0.72 \ 0.78 \ 0.81 \ 0.84$, for double riveting. (76)

Evidently for a given thickness of plate the efficiency of the plates increases with the diameter of the rivet, and consequently with p or p'—the pitch of the rivets. In boiler work small pitches are favorable to staunch joints.

THE PUNCHING AND DRILLING OF BOILER PLATES.

Punching is apt to tear and otherwise injure the iron in the neighborhood of the holes, particularly when the iron is poor; the spacing of the rivet holes is also apt to be inaccurate when they are punched. The injury to the metal about the holes, due to the excessive compression while being punched, can be partly removed by annealing the injured parts. In some of the best boiler shops, all these objections are met by reaming the punched holes after the plates are in position in the boiler.

Drilling is more expensive than punching, but is accompanied by greater accuracy in spacing and by greater strength of joint, which latter is due to the sounder condition of the metal near the holes. The sharp edges left by the drill should be rounded off to prevent their cutting into the rivets.

MACHINE RIVETING *versus* HAND RIVETING.

Machine riveting causes the rivet to fill the hole more perfectly, but is more liable to form the head excentrically to the rivet than hand riveting. Experiment serves to show that the machine-riveted joint is stronger than the hand riveted one, which is probably owing to the greater friction of the former.

For a discussion of the stresses to which boiler plates are subjected, see Unwin, Chap. IV. For a good discussion of the theory of riveted joints, see Weyrauch's "Structures of Iron and Steel" (Prof. A. J. Dubois' translation), pp. 550-560.

For a concise statement of the experiments made on riveted joints, see D. K. Clark's "Manual, etc., for Mechanical Engineers," pp. 630-643.

In estimating the probable cost of boilers it is necessary to know, among other things, the weight of the plates and rivets. The weight of the plates may be found by means of the table in Nystrom, pp. 359-361. The table given on the next page will serve to determine the weight of the rivets.

Iron Rivets.

Weight per 100 in Pounds.

Length under Head.	Diameters.						
	¼	⅜	½	⅝	¾	⅞	1
1	1.895	4 848	9.66	16.79	26.49	39.30	55.2
1⅛	2.067	5.235	10.34	17.86	27.99	41.4	57.9
1¼	2.238	5.616	11.04	18.96	29.61	43.5	60.7
1⅜	2.410	6.003	11.93	20.03	31.13	45.6	63.4
1½	2.582	6.402	12.43	21.04	32.74	47.8	66.2
1⅝	2.754	6.789	13.12	22.11	34.25	49.9	68.9
1¾	2.926	7.199	13.81	23.21	35.86	52.0	71.7
1⅞	3.098	7.566	14.50	24.28	37.37	54.1	74.4
2	3.269	7.956	15.19	25.48	38.96	56.3	77.2
2⅛	3.441	8.343	15.88	26.56	40.41	58.4	79.9
2¼	3.613	8.733	16.57	27.65	42.11	60.5	82.7
2⅜	3.785	9.120	17.26	28.73	43.67	62.6	85.4
2½	3.957	9.511	17.95	29.82	45.24	64.8	88.2
2⅝	4.129	9.898	18.64	30.90	46.80	66.9	90.9
2¾	4.301	10.29	19.33	31.99	48.36	69.0	93.7
2⅞	4.473	10.67	20.02	33.08	49.92	71.1	96.4
3	4.644	11.06	20.71	34.18	51.49	73.3	99.2
3⅛	4.816	11.44	21.40	35.27	53.05	75.4	101.9
3¼	4.988	11.84	22.09	36.35	54.61	77.5	104.7
3⅜	5.160	12.23	22.78	37.44	56.17	79.6	107.4
3½	5.332	12.62	23.48	38.52	57.74	81.8	110.2
3⅝	5.504	13.01	24.17	39.60	59.30	83.9	112.9
3¾	5.676	13.39	24.86	40.69	60.86	86.0	116.7
3⅞	5.848	13.78	25.55	41.78	62.42	88.1	119.4
4	6.019	14.17	26.24	42.87	63.99	90.3	121.2
4⅛	6.191	14.56	26.93	43.94	65.55	92.4	123.9
4¼	6.363	14.95	27.62	45.01	67.11	94.5	126.6
Wt. of 100H'ds.	0.519	1.74	4.14	8.10	13.99	22.27	33.15

Length of rivet required to make one head = 1½ diameters of round bar.

PLATE IV.

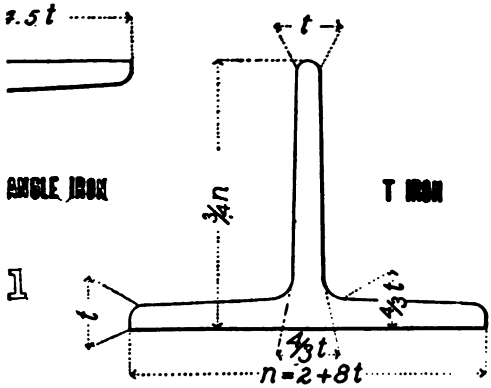
FASTENINGS.

d = diameter of rivet.
d' = diameter of stay-rod.
d_1 = diameter of split-pin for stay-rod.
d_2 = diameter of head of pin for stay-rod.
D = distance between screw-stays.
E = modulus of elasticity = 29,000,000 for iron.
f = permissible working stress in material; pounds per □″
p = pressure per □″ on surface of plate.
r = radius of a flat plate supported or fixed at circular edge.
r_0 = radius of flue or tube, also radius of nut or washer used for fastening stay-rod to flue-sheet.
t = thickness of shell.
t_0 = thickness of boiler-head or flue-sheet.
α = angle made by stay-rod with direction of pressure.
β = greatest deflection of circular plate supported or fixed at edge.

JOINTS FOR CONNECTING PARALLEL PLATES AND PLATES NOT IN THE SAME PLANE.

Fig. 1.—*Angle Iron.*

It is used for connecting plates which are not in the same plane and for stiffening plates against flexure. The thickness of either of the plates is represented by t. The proportions of the angle iron vary considerably, see Reuleaux, p. 175, Fig. 162, French Ed.; also pp. 172–173, German Ed., and Unwin, pp.



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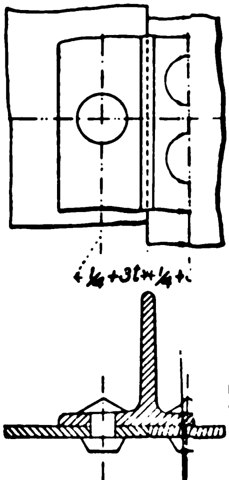


3

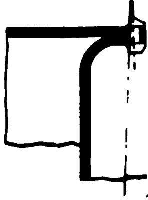
FASTENINGS.

ITS FOR CONNECTING PARALLEL LATE

AND

PLATES NOT IN THE SAME PLANE.

IV



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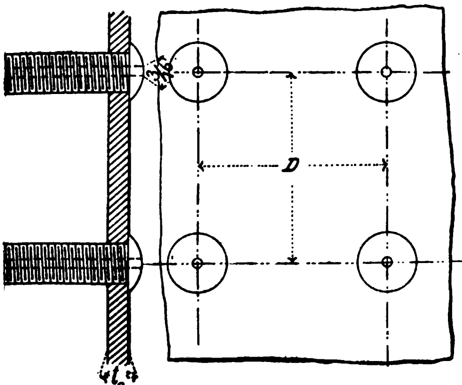

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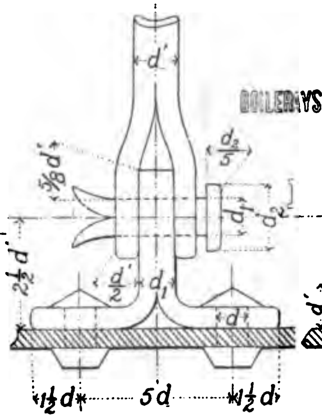
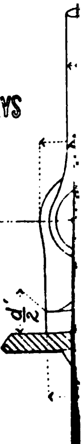




126-130. For weight per foot, see catalogues of the iron companies.

The angle iron receives its shape by being pressed, while red-hot, through suitably formed rolls.

Assume $t = \frac{1}{4}$ inch and draw to scale $\frac{1}{1}$ (full size).

Fig. 2.—*Tee Iron.*

It is principally used for stiffening plates against flexure. See Unwin, Fig. 62, p. 127, Reuleaux, p. 175, Fig. 163, French Ed., Fig. 166, p. 171, German Ed., and trade catalogues.

Assume $t = \frac{1}{4}$ inch and scale $= \frac{1}{1}$.

Fig. 3.—*Ring of Tee Iron, for Stiffening Flue Exposed to an External Collapsing Pressure.*

For the meaning of the symbols in the following example see page 13.

PROBLEM.—Given $D = 45$ inches, $D_1 = 15$ inches, $L_1 = 28$ feet, $P = 40$ pounds (gauge pressure) and $t = t_1$.

Required, t_1, l, n, also the factor of safety of flue when its plates have corroded $\frac{1}{8}$ of an inch.

UNSTAYED FLAT SURFACES.

For the case, Fig. *a*, of an imperforated circular plate supported at the edges, Grashof (Festigkeitslehre, pp. 236- 237), finds the two following formulas:

$$t_0 = r\sqrt{\frac{5p_1}{6f}} \qquad (77)$$

$$\frac{\beta}{t_0} = \frac{2}{3}\left(\frac{r}{t_0}\right)^4 \frac{p}{E}. \qquad (78)$$

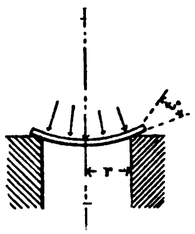


Fig. a.

For the case of an imperforated plate whose edges are fixed between two ring plates or rigidly held as shown in Fig. *b*, Grashof also gives the two formulas:

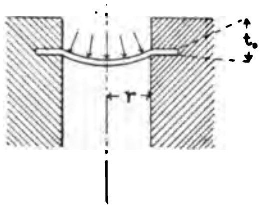


Fig. b.

$$t_0 = r\sqrt{\frac{2p}{3f}}, \qquad (79)$$

$$\frac{\beta}{t_0} = \frac{1}{6}\left(\frac{r}{t_0}\right)^4 \frac{p}{E} \qquad (80)$$

Supposing these formulas to be applicable to the unstayed flat and circular plates constituting the ends or heads of boilers Grashof finds that the ratio $\frac{t_0}{t}$ of their thickness to that of the boiler shell,

$$\begin{array}{llllll} \text{when } p = & 30 & 60 & 90 & 120 & 150 \text{ pounds.} \\ \text{is } \frac{t_0}{t} = & 6.5 & 5.2 & 4.5 & 4.0 & 3.6. \end{array} \qquad (81)$$

This would give a thickness of boiler head altogether too great, consequently flat boiler plates should never be used unless stayed by flues, tubes, or stay-rods.

The above formulas are only applicable within the elastic limits of the material used.

Figs. 4, 5, 6 and 7.

In these figures are given the different methods of fastening the head of the boiler to its shell. Experiments made by Robert Wilson in England (detailed in *Engineering*, September 28, 1877, p. 239), appear to show that the mode of attaching by flange or by an inside or outside angle iron exerts an important influence on the manner in which the plate is strained by the pressure. Attaching by outside angle iron was found to be decidedly the weakest of the various modes of fastening tried.

Attaching by inside angle iron, Fig. 5, was found to have greater strength than a flat flanged plate similar to that in Fig. 4.

The strongest mode of fastening was one resembling Fig. 6, for this did not even take an observable permanent set at pressure which broke the heads used in the other modes of fastenings. It should be remarked, however, that the strongest form instead of being flat across its central portion, as in Fig. 6, differed from the latter in being dished to a radius of 3 feet. These experiments also showed, that, though the bursting pressure was in all cases quite high, deflection of the plates occurred almost as soon as any pressure was put upon them, and they sprang back again on the pressure being taken off. "This springing of the plate and compression at the root of the flange when the pressure is alternately applied and removed in actual work, in the course of time inevitably results in grooving or channeling, which, especially when aided by the action of the corrosive acids in the water or steam, will in time reduce the thickness of the plate to a knife edge, and bring about the destruction of an unstayed surface at a very low pressure." Tightness of joint is always an important consideration; in this respect, the joints shown in Figs. 4 and 6 are probably better than those of Figs. 5 and 7.

When $t =$ or $< \frac{1}{2}$, thickness of boiler head is often taken $t_o = 1\frac{1}{2}t$.

In Figs. 4 and 5 assume $t = \frac{1}{4}$ inch and draw full size.

In shading Figs. 6 and 7 it should be noticed that the light portion is on the side from which the light is supposed to come.

Assume $t = \frac{5}{16}$ inch and scale $= \frac{1}{4}$.

Fig. 8.—*Wrought Iron Ring for Furnace Opening.*

Wrought iron is greatly to be preferred to cast iron for the rectangular ring surrounding the entrance to the furnace, because the boiler plates are also of wrought iron, and there is, therefore, less tendency to crack the joints than if the latter were composed of unlike material which expanded unequally.

Assume $t = \frac{5}{16}$ inch and scale $= \frac{1}{4}$.

Fig. 9.—*Joint for Connecting Parallel Plates.*

The rectangular bar separating the plates is of cast iron and may be used in the coldest portions of the boiler; it is thus often used at the bottom of a boiler to connect the flat plates which enclose a water space. Channel (wrought) iron is also used for the same purpose, but is not well suited for forming corners.

Assume $t = \frac{5}{16}$ inch, $d = 2t$, scale $\frac{1}{4}$.

STAYED, FLAT, SURFACES.

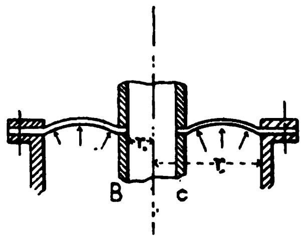


Fig. c.

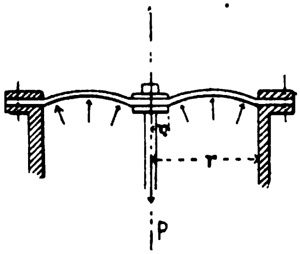


Fig. d.

Fig. *c* represents the case in which a flat circular plate is perforated at the center, its outer and inner edges, being rigidly fixed between ring plates. Grashof finds that the greatest strain occurs at the inner edge, and, other things being equal, that the strain increases when $\frac{r_0}{r}$ diminishes.

Assuming that this case is like that of the boiler head or flue sheet whose outer rim is riveted to the boiler shell by means of angle iron, and whose inner edge or rim is likewise fastened to an internal fire tube, by means of angle iron, Grashof finds that when $\frac{r_0}{r} = \frac{1}{2}$, the permissible working stress f is only $\frac{1}{6}$ of that of the imperforated plate in Fig. *b*, and when the tube BC of Fig. *c* is eccentric (as is usually the case in flues), that f may be

assumed as only $\frac{1}{4}$ of that allowable in the imperforated plate of Fig. *b*. From this it follows that the ratio $\frac{t_0}{t}$ of the thickness t_0 of the boiler head or flue sheet to the thickness t of the shell :

when $p = 30 \quad 60 \quad 90 \quad 120 \quad 150$ pounds per □″ (gauge),

is $\frac{t_0}{t} \geqq 3.2 \quad 2.6 \quad 2.2 \quad 2.0 \quad 1.8.$ (82)

As it is usual to make t_0 but slightly greater than t (from $1\frac{1}{8}t$ to $1\frac{1}{2}t$) the foregoing result shows that it is necessary to stiffen the two flue sheets, either by connecting them with each other by means of stays, or—what is still better—by connecting them with the shell of the boiler by means of radial stays. For a single central stay, as in Fig. *d*, Grashof finds for the load P to which it is subjected,

$$P = \frac{\mu}{4}\pi r^2 p, \tag{83}$$

for $\frac{r_0}{r} = 0 \quad 0.1 \quad 0.2 \quad 0.3,$

$\mu = 1 \quad 1.17 \quad 1.39 \quad 1.67.$

$P = 0.79 \quad 0.92 \quad 1.09 \quad 1.31 . r^2 p,$ (84)

from which the cross section of the stay may be determined. One stay will seldom be found sufficient, and when several are used the rule given below (under Fig. 11) may be followed.

Thickness of Flat Stayed Surfaces.

When the plate is held at points which divide it into squares whose sides each equal D and is loaded by the specific pressure p we will have,

$$t_0 = D\sqrt{\frac{2}{9}\frac{p}{f}} \tag{85}$$

$$t_0 = D\sqrt[3]{\frac{1}{36}\frac{D}{\beta}\frac{p}{E}} \tag{86}$$

When boiler is supplied with fresh water take $f = 3000$ pounds for iron, then,

$$t_0 = \tfrac{7}{800} D\sqrt{p}. \tag{87}$$

When supplied with salt water take $f = 2300$ pounds for iron, then,

$$t_0 = 0.01\ D\sqrt{p}. \tag{88}$$

The greater value of t_0 makes allowance for the greater corrosion, and the smaller one corresponds to a factor of safety (against bulging) of from 4 to 5.

Fig. 10.—*Screwed Stays.*

These are much used for connecting parallel plates. In the present figure the inner plate is of copper, which was formerly much used for locomotive fire-boxes. In such cases (still occurring) $t_1 = 1\frac{1}{4}t$ and $d = 2t$, t being the thickness of the equivalent or outer iron plate.

As these screwed stays are liable to break across the threads inside the plates, and such fractures would bring an increased pressure on the adjacent stays, it is important that warning should be given of such a fracture; this is accomplished by boring $\frac{1}{8}$ or $\frac{3}{16}$ of an inch hole in the center of the stay, which will allow water or steam to escape when the stay is broken. These stays are cut from long screws, square at one end, a length of $1.8d$ being allowed for forming the head of the stay.

Assume $D = 4\frac{1}{4}$, $p = 150$ pounds (gauge pressure in locomotive boiler).

Fig. 11.—*Boiler Stays with Crow Feet.*

Let D^2 = area supported by each stay,

then will $$d' = \frac{D\sqrt{p}}{80} \tag{89}$$

for fresh water,

and $$d' = .014\ D\sqrt{p} \tag{90}$$

for salt water.

These formulas correspond to a high factor of safety—8—allowance being made for the welding of the edges to the stay, and for corrosion.

$$d = \tfrac{1}{2}d' + \tfrac{3}{16}, \tag{91}$$

$$d_1 = 0.8d', \tag{92}$$

$$d_2 = 1\tfrac{1}{2}d_1 = 1.2d'. \tag{93}$$

On diagonal stays the tension on stays equals

$$\frac{pD^2}{\cos\alpha}, \tag{94}$$

α being the angle made by the stay with the direction of pressure. The stays are brought into tension by being pinned to their crow feet while red hot, the contraction of the rods while cooling being sufficient to tighten them.

Assume $p = 60$ pounds (gauge), $D = 10$ inches, scale $= \frac{1}{2}$.

Fig. 12.—*Joint for Connecting Parallel or Concentric Plates.*

Assume $t = \frac{1}{4}$ inch. Scale $= \frac{1}{2}$.

Fig. 13.—*Socket Rivet.*

This serves to unite the parallel plates of a water space. The socket is formed of a piece of boiler plate and serves to resist the external pressure of the atmosphere whenever there is a vacuum in the boiler; otherwise there would be a tendency to collapse. The screwed stays of Fig. 10 resist both internal and external pressure.

$$d = 2t \text{ to } 2\tfrac{1}{2}t. \tag{95}$$

Assume $t = \frac{5}{16}$ inch, scale $= \frac{1}{2}$.

Other forms of joints are shown in Reuleaux, p. 175, French Ed., and p. 172, German Ed.; also Unwin, p. 128.

PLATE V.

FASTENINGS.

b = width of key.
d = diameter of shaft.
d' = diameter of set screw.
D = external diameter of hub or socket.
n = number of revolutions per minute of shaft.
N = number of horse-powers transmitted.
PR = twisting moment of shaft in pound-inches.
R = radius of pulley.
t = average thickness of key.
u = radius of key-boss for strengthening key-way.
w = thickness of material around eye of wheel.
W = width of pulley.
β = breadth of strap.
δ = thickness of strap.

Keys for Pulleys, Wheels, Cranks, Etc.

Gibs and Cotters for Connecting Strap with Rod.

Formulas for Keys.

When the force transmitted is small, we have,

$$b = \frac{d}{7} + \tfrac{1}{4}, \tag{96}$$

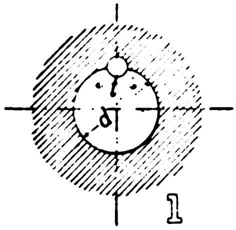
$$t = \frac{d}{12} + 0.16, \tag{97}$$

When the force transmitted is large, the shaft being subjected to a considerable strain of torsion, we have,

$$b = \frac{d}{5} + 0.16, \tag{98}$$

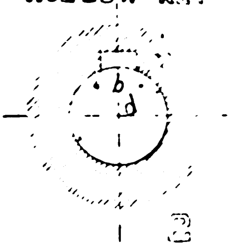
$$t = \frac{d}{10} + 0.16, \tag{99}$$

ROUND KEY



1

HOLLOW KE



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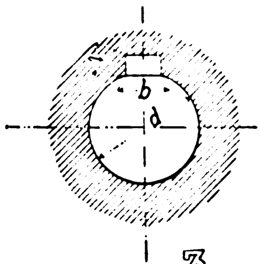
FLAT KEY



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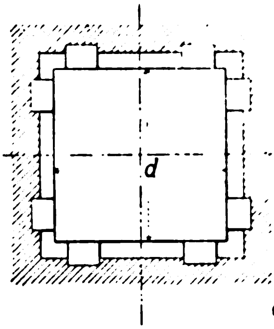
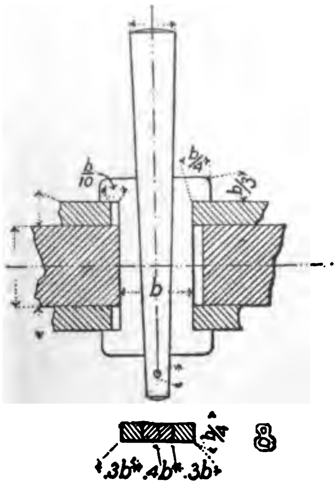


4

KEYS FOR

GIE



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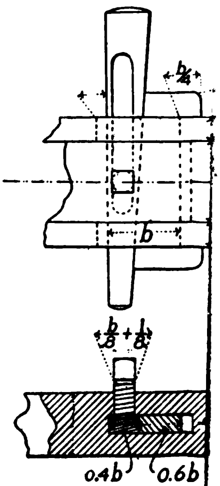


'hen $d =$ or < 1.25, $b = \frac{d}{3}$, (100)

and $t = \frac{d}{5}$ (101)

When the pulley is placed on a very large shaft the dimensions the key are referred to the thickness of the metal around the re of the wheel, and we have,

$$t = \frac{w}{5} + \frac{1}{8}, \tag{102}$$

$$b = \frac{5}{3}t. \tag{103}$$

Or we may take for the unit of reference,

$$d = 4\sqrt[5]{\frac{N}{n}}, \tag{104}$$

or $$d = 0.1 \sqrt[3]{PR}, \tag{105}$$

where $N =$ number of horse-powers transmitted by this pulley and n the number of revolutions per minute of the shaft.

Taper of key $= \frac{1}{100}$, or $\frac{1}{8}$ inch to the foot, the taper is put into the hub or boss, the key-way in the shaft being parallel to the axis of the shaft. Place on the drawing of the key its thickness at the large end.

Fig. 1.—*Round Key.*

This key is often used for fastening hand wheels, and since it is usually meant to be a permanent fastening it has no taper or only a very slight one, being made to fit tightly. Assume

$$d = 1, \text{ and } t = \frac{d}{12} + 0.16.$$

Fig. 2.—*Hollow Key.*

This is used when the force transmitted is small, as in light pulleys. It allows the pulley to be shifted to another position on the shaft without preparing a key-bed. It is also used with a

sunk key, being placed at right angles to the latter, and then not only gives greater security against slipping, but also prevents rocking. It is used in a similar manner with set screws. Its resistance to slipping is of course due to friction only.

Assume $d = 1\frac{1}{8}$.

Fig. 3.—*Flat Key.*

This key is also to be used where light stresses are to be transmitted, but it requires that the part of the shaft to which the piece is to be fastened shall first be flattened with a file. It is capable of transmitting a greater force than the hollow key.

Take $d = 1\frac{1}{2}$ inches.

Fig. 4.—*Flat Key for Square (unfinished) Shafts transmitting Great Stresses.*

By varying the thickness of the keys the wheel can be easily centered. This mode of securing wheel to shaft is very reliable, since each key has considerable of a lever arm for resisting the twisting moment or slipping tendency. Where a number of keys are employed, as in the present case, their dimensions may be taken as if a small force only were to be transmitted.

Take $d = 4$ inches.

Fig. 5.—*Sunk Key.*

This is the most useful and trustworthy form of key for fastening machine parts to a round shaft when heavy stresses are to be transmitted. The boss of the wheel is not always strengthened, as shown in the figure, but it is well to do so whenever the stresses are great, as there is a strong tendency for the boss of the wheel to fracture at the sharp corners of the key-way. The boss may be also strengthened by shrinking on rings of wrought iron of a thickness $\frac{w}{2}$ = half thickness of metal around eye of wheel. The radius of key-boss for strengthening the key-way may be taken at $u = w + \frac{1}{4}$ inch. (106)

When the key can not be driven out from the smaller end a gib or head is given to it. The key-way in the shaft should be of sufficient length to readily drive the key in and out of position.

Take $d = 2$ inches and length of hub $= 3$ inches.

Fig. 6.—*Set Screw Fastenings.*

These set screws have cup-shaped points, which cause less indentation in the shaft than the ordinary set screw. They are also used when the force to be transmitted is but slight. A hollow key sometimes accompanies these screws. Let $W =$ width of pulley and $R =$ its radius. Then the emprical formula for diameter of set screws is as follows:

$$d' = 0.003WR + \tfrac{3}{8}. \qquad (107)$$

Assume $W = 7\tfrac{1}{2}$ inches, $R = 7$ inches, Scale $= \tfrac{1}{2}$.

Fig. 7.—*Key or Cotter for Uniting a Rod to a Cast Iron Socket*

$D =$ external diameter of socket $= 2d$. Taper $= \tfrac{1}{4}$ inch in 5 inches.

Assume $d = 2$ inches.

Fig. 8.—*Key and Gibs for Uniting Straps with Rod.*

When $\beta =$ breadth and $\delta =$ thickness of straps, the combined section of key and gibs is about $1\tfrac{1}{4}\beta\delta$, provided strap, key and gibs are of the same material.

When the key is intended as a permanent fastening the taper may have any value from $\tfrac{1}{30}$ to $\tfrac{1}{100}$.

When the key is to be often taken out, taper $= \tfrac{1}{16}$ to $\tfrac{1}{24}$.

When the key is secured, taper $\geqq \tfrac{1}{16}$. A steel key or cotter may have $\tfrac{3}{4}$ the breadth of an iron one, the other dimensions remaining the same. The clearance or draught should be at least large enough to permit the key being driven in till its top is in line with the ends of the gibs. The hooks in the gibs are for the purpose of preventing the straps from spreading.

Let depth of rod $= 3\tfrac{1}{4}$ inches.

Assume taper $\tfrac{1}{24}$. $\beta = 3$ inches, $\delta = \tfrac{3}{4}$ inch.

Fig. 9.—*Single Gib and Key for Uniting Strap with Rod. Key Secured by Set Screw.*

Leading dimensions as in Fig. 8. Taper $= \frac{1}{16}$.

Set-screw groove must be parallel to vertical edge of key.

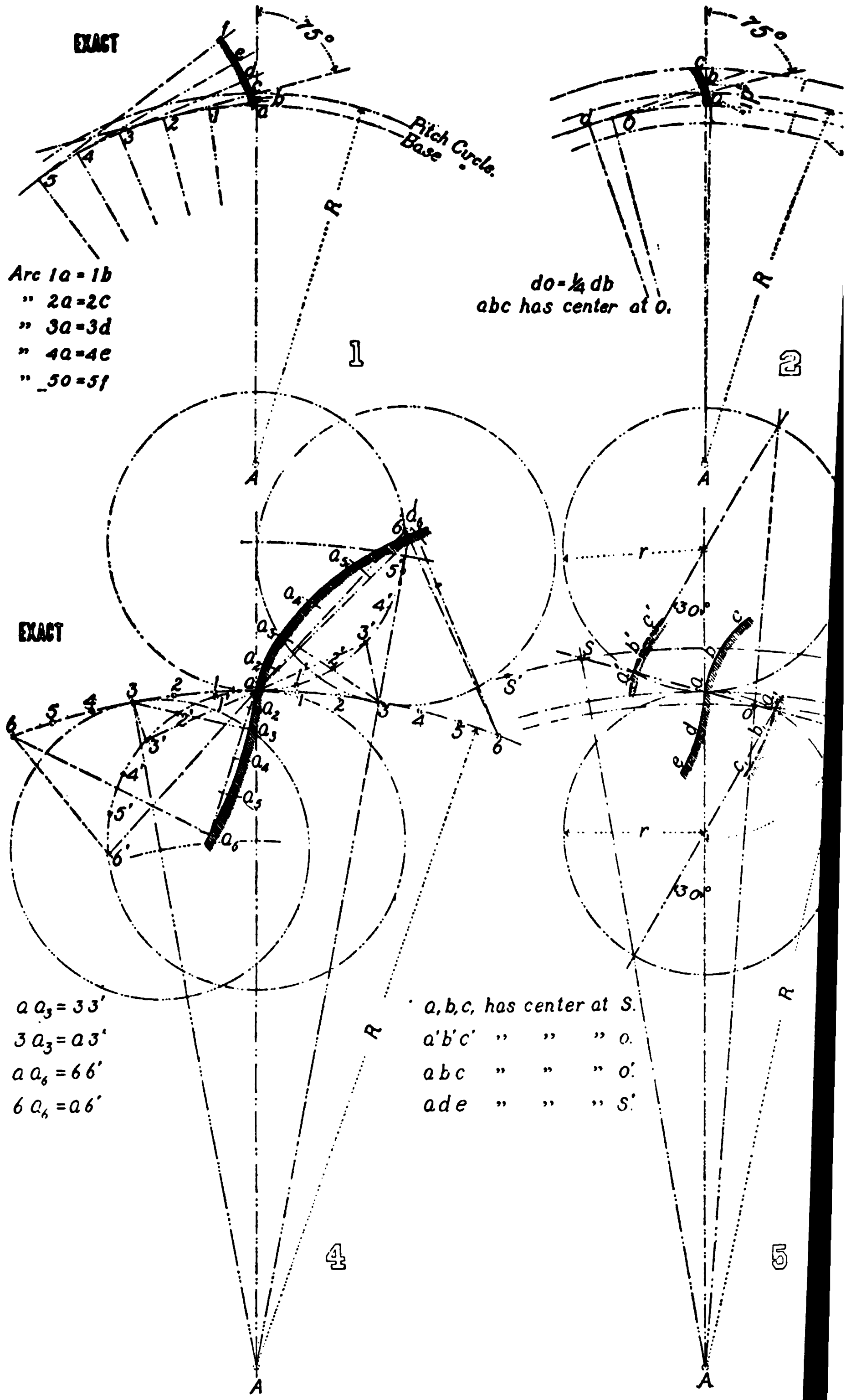
Fig. 10.—*Key Secured to Gib by Screwed Prolongation of Gib or Key.*

Assume taper $= \frac{1}{8}$. Dimensions and scale as in Fig. 9.

Fig. 11.—*Key or Cotter for Uniting One Rod to Another with Socket-Shaped End.*

$D = 2d$ for both wrought and cast iron. Taper $\frac{1}{24}$. $d = 2$ inches.

For other forms of keys and methods of securing them see Reauleaux, pp. 190-201, French Ed.; pp. 184-195, German Ed.; also Unwin, pp. 166-180.


EXACT
75°
Pitch Circle.
Base
Arc 1a = 1b
" 2a = 2c
" 3a = 3d
" 4a = 4e
" 5a = 5f
1
75°
do = ¼ db
abc has center at o.
2
EXACT
$a a_3 = 3 3'$
$3 a_3 = a 3'$
$a a_6 = 6 6'$
$6 a_6 = a 6'$
4
a, b, c, has center at S.
a'b'c' " " " o.
abc " " " o'.
ade " " " S'.
5

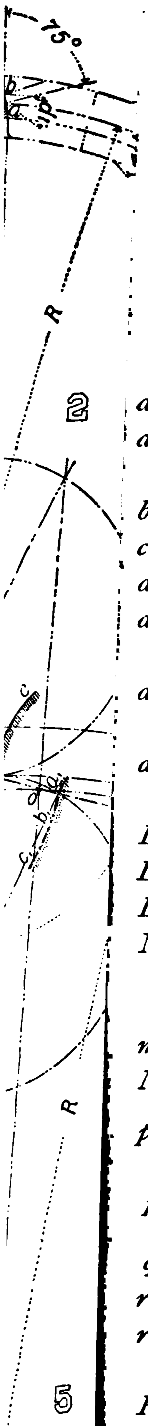


PLATE VI.

GEARING.

a and a' are the major axes of two confocal ellipses; $a > a'$.

a_1 and b_1 = sin β and cos β respectively; they enter as coefficients into formulas for m and n.

b and b' are the minor axes of two confocal ellipses; $b > b'$.

c = backlash divided by circumferential pitch.

d = diameter of any describing circle.

d_0 = diameter of describing circle for set of interchangeable wheels.

d_1 = diameter of describing circle for face of D_1 and flank of D_2.

d_2 = diameter of describing circle for face of D_2 and flank of D_1.

D = nominal diameter of wheel = diameter of pitch circle.

D_0 = diameter of base circle for involute teeth.

D_1 and D_2 = diameter of the pitch circle of a *pair* of wheels.

M_1 and N_1 = portions of pitch diameter representing coordinates CO and CD for locating tangential reference line to pitch circle at pitch point.

m and n = same when word *base* is substituted for *pitch*.

N = number of teeth in smallest wheel of the set.

$$p' = \text{circular pitch} = \frac{\text{circumference of pitch circle of wheel.}}{\text{number of teeth in wheel.}}$$

$$p = \text{diametral pitch} = \frac{\text{number of teeth in wheel}}{\text{diameter of wheel.}}.$$

q = abbreviation for expression $2a_1M_1 + 2b_1\sqrt{0.25 - M_1^2}$.

r = radius of describing circle.

r_0 = radius of describing circle for set of interchangeable wheels.

R = nominal radius of wheel = radius of pitch circle.

s = convenient multiple of coordinate N or n; used in locating reference line.

x and y = coordinates of points on the profile of a tooth.

x' and y' = ditto after transformation of coordinates.

α = angle made by the generatrix of involute profiles with line of centers.

β = angle included between the radii passing through the starting point and the pitch point of involute.

δ = angle FST (see Fig. p. 41) made by the third secondary centrode KG or FH with the tangent ST to the two primary centrodes.

ϑ = angle subtending the arc of the base circle rectified during the generation of the involute.

γ = angle included between axis of ordinate and center line of tooth space when origin is on base circle.

θ = angle included between center line of tooth space and radius through pitch point of tooth profile.

ω = angle through which the describing circle has rolled since the generating point left the centrode.

Methods of Drawing the Curves of Teeth.

The determination of the curves of teeth is a particular case of the general problem of finding the profiles of elements for a given motion.

The various methods of solving this problem are given by Reuleaux in "Kinematics of Machinery," pp. 146-166,

The principal kinds of toothed gearing are as follows:

Spur gearing, in which the axes of the wheels are parallel.

Bevel gearing, in which the axes of the wheels are inclined and intersect.

Skew gearing, in which the axes of the wheels are inclined and do not intersect.

In the most frequently occuring examples of these the relative motion of a pair of wheels is such that *the ratio of their angular velocities is constant;* it is only in comparatively rare cases—and then generally as a species of spur gearing (eliptical wheels for example) that the velocity ratio is variable.

Constant velocity ratio in the case of spur, bevel and skew wheels, is represented respectively by cylindrical, conical and hyperboloidal axoids having circular bases. Variable velocity ratio in the case of spur wheels is represented by cylindrical axoids having *non-circular* bases.

Whenever the axoids are cylindrical the profiles can be readily obtained from the centrodes, as explained by Reuleaux, pp. 146-166; but when the axoids are not cylindrical, although the profiles may be obtained in a manner analogous to that employed for the cylindrical ones, the problem is nevertheless much more complicated, as it involves space of three dimensions.

This complication can be avoided whenever the outline of any section of a tooth can be found from plane curves, which, though not centrodes, are employed as such.

We will hereafter show that such plane curves are easily found for bevel and skew wheels, and that the resulting profiles are sufficiently accurate for the former, and absolutely so for the latter.

The finding of the curves of teeth in all the important kinds of gearing is therefore reducible to that of spur gearing; that is, to the case of conplane motion represented by centrodes.

The various methods given by Reuleaux ("Kinematics of Machinery," pp. 146-166) are therefore directly applicable to the determination of these curves.

Of the various methods, those most frequently employed are:

Auxiliary Centrodes, Secondary Centrodes, Equidistants and Approximations by means of circular arcs. The first method given by Reuleaux has recently been directly applied by M. A. Swasey to the cutting of spur wheel teeth by circular cutters. See Vol. XII, Trans. Amer. Soc. M. E.

As the Equidistants and Approximations are deduced from the first two, the relative advantages and disadvantages of these are of the greatest importance. Before comparing them, however, we will show how the required curves can be obtained by their means.

Methods of Finding the Curves of Teeth by Secondary Centrodes.

Fig. 1.—*Exact Method of Finding the Curves of Involute Teeth.*

If an extensible thread which has been wrapped around a given plane curve be unwound, a pencil attached to the free end of the taut thread will describe a curve called the involute of the given curve. Teeth profiled with this curve are called involute teeth. When the primary centrode is a circle the secondary centrode is also a circle, the two centrodes being respectively known in practice as the pitch and base circles of involute teeth.

The base circles usually emyloyed can easily be obtained by drawing through any assumed point of the pitch circle a line making an angle of 15° with this circle, this line will then be tangent to the base circle, and the perpendicular let fall from the center of the base circle on to this tangent will be the required radius.

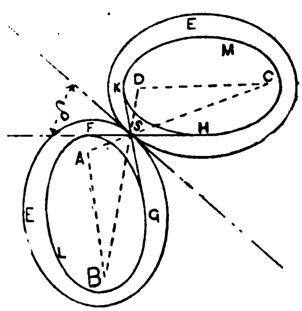
If the assumed point of the pitch circle is on the line of centers of two wheels, the tangent to the base circle is said to make an angle of 75° with the line of centers. This angle can be easily laid off by means of the 45° and 30° triangles used by draughtsmen.

Smaller angles than 75° would cause greater pressure on the bearings of the wheel shafts, and are therefore not employed.

If the initial point a of the involute is given we can construct the involute curve as follows: Starting from the point a divide the base circle into any convenient number of equal parts, $a1$, 12

etc.; at the points of division draw tangents and rectify on each tangent the arcs included between its point of tangency and the initial point a, the line passing through the outer ends of the rectified arcs will be the required involute curve.

In the case of elliptical wheels the primary and secondary centrodes belonging to each wheel are confocal ellipses, for example, the ellipses E and L have the foci A and B in common, while E and M have C and D in common. Since the pitch ellipses or primary centrodes E and E' are equal, the base ellipses or secondary centrodes LGF and KHM are also equal. If the wheels are set properly the distance BD will always equal the distance between the centers A and C of the wheel shafts, and the point of contact of the pitch ellipses will fall on the intersection of BD and AC.



The third secondary centrode KG or FH makes an angle $FST = \delta$ with the primary centrodes.

Representing the major and minor axes of the confocal ellipses respectively by a, a' and b, b', we have for the limits between which the angle δ varies, the two expressions,

$$\cos \delta = \frac{a'}{a} \text{ and } \cos \delta = \frac{b'}{b}. \tag{108}$$

Involute curves suitable for the teeth of the elliptical wheels can be obtained by unwinding the line SG from the base ellipse FGL, the process of finding the points of the involute curve being exactly analogous to that employed for the circular wheels.

It would be well (by means of the above formulas) to so choose the axes of the confocal ellipses that the average value of

the angle δ should not exceed 15°, the pressures on the bearings of the shafts will then be the same as in similarly circumstanced circular wheels.

The application of involute teeth to elliptical wheels is discussed in "Der Civil Ingenieur," 1875, p. 223.

The student should remember in applying this method to other than circular wheels, that, although Reuleaux is right in saying that secondary centrodes are universally applicable for determining profiles, he is wrong in saying that these secondary centrodes are the envelopes of secants making a constant angle with the primary centrodes. This holds when the wheels are circular, but does *not* hold (as we have just seen) when the wheels are elliptical.

METHOD OF FINDING THE CURVES OF TEETH BY AUXILIARY CENTRODES OR DESCRIBING CIRCLES.

Fig. 4.—*Exact Method of Finding the Curves of Epicycloidal Teeth.*

If a circle be rolled on the outside of another circle, any point on the circumference of the first will describe an epicycloidal curve. If a circle be rolled on the *inside* of another circle, any point of the circumference of the rolling circle will describe a hypocycloidal curve. Teeth whose profiles are formed by these two curves are called epicycloidal teeth. Generally the epicycloidal portion of the profile is called the *face* of the tooth, and the hypocycloidal portion the *flank* of the tooth; the only exception is in the case of annular wheels where the designations are reversed. In all cases the flank is nearest the base or root of the tooth.

If the circle on which the rolling takes place is a pitch circle or primary centrode, the rolling circles—properly called describing circles—are evidently auxiliary centrodes, capable of describ-

ing correct profiles for the toothed elements; a necessary consequence is that *the same describing circle must be used for those parts of the teeth which work together*, that is, the *faces* of the teeth on one wheel of a pair must be generated by the same describing circle which generates the *flanks* of the other wheel.

CHOICE OF DESCRIBING CIRCLES.

The magnitude of the describing circle varies according to the conditions imposed upon the teeth and their wheels with respect to cost, form, strength, durability and freedom from frictional resistances.

When toothed wheels are to be cast by means of patterns it is of economical importance that any two of a set of wheels having the same pitch should be capable of working correctly with each other; this condition requires that the form of a tooth should be independent of the diameter of any wheel of the set except its own; by employing the same sized describing circle for every one of its epicycloidal and hypocycloidal curves belonging to this set of wheels, this condition is satisfied. The size of the describing circle usually chosen for a set of interchangeable wheels is one whose diameter is half that of the wheel having the smallest number of teeth occurring in the set; and, as the size of this wheel depends only upon the size of its teeth, we have for the radius of the describing circle

$$r_0 = \frac{d_0}{2} = \frac{1}{2} \times \frac{Np'}{2\pi} = \frac{Np'}{4\pi} = \frac{N}{4p}, \qquad (109)$$

d_0 representing the diameter of the describing circle,
N representing the number of teeth in smallest wheel of the set,

$$p' \text{ representing circular pitch} = \frac{\text{circumference of any wheel}}{\text{No. teeth in the wheel}}, \qquad (110)$$

$$p \text{ representing diametral pitch} = \frac{\text{No. teeth in wheel}}{\text{diameter of wheel}} = \frac{\pi}{p'}, \qquad (111)$$

In this country and in England $N = 12$, then,

$$r_0 = \frac{p'}{1.0072} = 0.955p' = \frac{3}{p}. \qquad (112)$$

When the wheels are machine molded, that is, formed from a pattern of two or three teeth only in a wheel molding machine, the economy of employing interchangeable wheels disappears, and the designer has then greater liberty in the choice of describing circles.

The form or profile of a tooth evidently depends only upon the diameter of its pitch circle and the diameters of the describing circles which generate its face and flank. If we represent by D_1 and D_2 the diameters of the pitch circles of a pair of wheels, by d_1 the diameter of the describing circle for both the face of D_1 and the flank of D_2, and by d_2 the diameters of the describing circles for the face of D_2 and the flank of D_1, we have for the usual limits of d_1 and d_2 the formulas,

$$d_1 \leqq \frac{D_2}{2} \qquad d_2 \leqq \frac{D_1}{2} \qquad (113)$$

When $d_1 = \frac{D_2}{2}$ and $d_2 = \frac{D_1}{2}$ the profiles of the teeth within each of the pitch circles will be radial lines (giving a simple form of tooth well suited for wooden teeth or cogs used with high speeds).

The face of a tooth in each wheel (having radial flanks) will then depend on the diameter of the other wheel of the pair, and will therefore not work correctly with any other size of wheel of the same pitch whose flanks are radial and whose faces have been generated by describing circles.

In comparatively rare cases $d_1 > \frac{D_2}{2}$. In such cases, however, the root of the tooth of D_2 is either weakened or completely cut away, so that it must be strengthened by a curve which will not interfere with the transmission of a constant velocity ratio.

The method of doing this is explained in Reuleaux's "Konstructeur," p. 632, French Ed. The extreme limits $d_1 = D_2$ and $d_1 = o$, correspond respectively to the Fourth and Seventh methods in "Kinematics of Machinery," pp. 157 and 164.

By taking $d_1 < \frac{D_2}{2}$ the flanks of the tooth diverge from its central line, giving a thick, strong root to the tooth.

The durability of a tooth, or its resistance to change of form due to the wear of friction, is also affected by the size of the describing circle. With large describing circles (other things being equal) more teeth are in contact and each individual tooth is consequently subjected to less pressure at the point of contact, but this favorable condition is accompanied and more than neutralized by a decrease in the length of the acting part of the flank, so that the final result of an increase of describing circle is to increase the change of form always taking place between the acting surfaces of teeth. Chief dependence against change of form should however be placed on great length of line of contact (this length is measured on a line parallel to the axis of wheel and is equal to breadth of wheel).

Fig. 2.—*Approximation to Involute Curve by Means of a Single Arc.*

This method to be employed when number of teeth = or < 60. Base circle is first described with a radius equal to the perpendicular distance of the 75° line from center of pitch circle. Then the point b is taken at a distance from addendum circle equal to one-third the working depth of the tooth. (The outer circle containing the tops of the teeth is called the addendum or tip circle, and the distance between the latter and the pitch circle is called the addendum of the curved tooth.) The working depth of a tooth is assumed to be equal to the sum of its own adden-

dum plus that of its partner; this sum ordinarily is equal to twice the tooth's own addendum, *i. e.*, $= 2 \times .3p' = .6p'$ or $= 2 \times \frac{1}{p} = \frac{2}{p}$. In the most commonly occurring cases, therefore, b is to be taken at $0.1p'$ or at $\frac{1}{3p}$ from pitch circle. Through the point b a tangent bd is drawn to base circle and at the point o of this tangent—at a distance from the point d of contact $= do = \frac{1}{4}db$—as a center, and with ob as a radius, we describe the arc cba. This arc is limited by the addendum and base circles. Inside the base circle the profile of the tooth consists of a portion of the radius of the wheel and a small circle just sufficient to round the sharp cusp which would otherwise be formed by radial portion of the profile and the root circle.

Assume 10″ wheel and No. 3 diametral pitch, scale $= \frac{1}{1}$.

Fig. 3.—*Willis' Odontograph for Involute Curves.*

The instrument should be only employed when the base circle falls inside of that circle which forms the inner limit of the working depth of the tooth, that is, it should only be used when the number of teeth of the wheel = or > 60.

When the single arc is used as an approximation,

$$ob = R \cos a. \qquad (114)$$

In Willis' instrument for single arcs $a = 75° \; 30'$, because cos $75° \; 30' = .25038 = \frac{1}{4}$ nearly. The odontograph consists, therefore of a bevel $75° \; 30'$ made of brass or card paper as in the figure of which the side ob is graduated into a scale of quarter inches and tenths. If this bevel be laid upon the radius bA, so that its point b coincides with the pitch circle, the center point o will be found at once by reading off the radius of the wheel in inches upon the reduced scale. Thus the radius bA of the figure is 5 inches long and the point o is found at the point 5 on the scale. Draw full size.

Fig. 5.—*Approximations to Epicycloidal and Hypocycloidal Curves by Means af Circular Arcs.*

In the figures the describing circles were asumed equal,

$$r_0 = \cdot 955p' = \frac{3}{p},$$

as in an ordinary, interchangeable, set of wheels, in which the diameter of the describing circle is equal to half the diameter of that wheel in the set which has 12 teeth. But the method here given holds good when the describing circles are unequal, when the wheel is an annular one, and also for Racks.

In this method radius of face is equal to,

$$ob' = 1.0353r_0 \times \frac{R + r_0}{R + 2r_0}, \qquad (115)$$

where r_0 = radius of describing circle for face, and R = radius of wheel. The radius of flank equals,

$$Sa_1 = 1.0353r_0 \times \frac{R - r_0}{R - 2r_0}, \qquad (116)$$

r_0 being radius of describing circle for flank.

Assume 18 inch wheel, No. 1 diam. pitch ($p = 1$). Scale = $\frac{1}{4}$.

Fig. 6.—*Approximation to Epi- and Hypo-cycloids by Means of the Willis Odontograph.*

Describe from the center A the arc of the given pitch circle, and upon it set off DE equal to the pitch (p') and bisected in a. For the curve within the pitch circle (flank of tooth) apply the slant edge of the instrument to the radial line AE, placing its extremity E on the pitch circle as in the figure. In the table headed *Centers of the Flanks of Teeth*, look down the column of the given circular pitch p' and opposite to the required number of teeth. The point s indicated in the drawing by the position of this number on the scale of equal parts (drawn on the long arm of the instrument) is the center required from which the arc *amn* must be drawn with the radius sa. The center for the arc

ade (or face) which lies outside the pitch circle, is formed in a manner precisely similar by applying the slant edge of the instrument to the radial line *AD*. In using the instrument it is only necessary to recollect that the portion of the scale employed, and the point *a* always lie upon opposite sides of the radial line to which the instrument is applied. The curve *edamn* is also correct for an annular wheel of the same diameter and number of teeth, *e* becoming the root and *n* the point or tip of the tooth.

Tables showing the Place of the Centers upon the Scales.

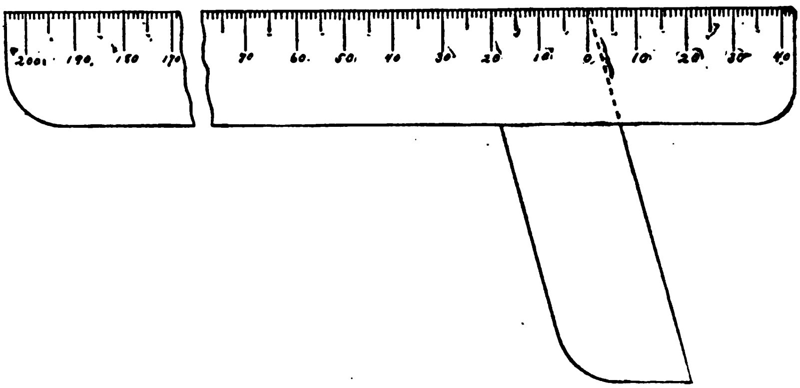
Centers for the Flanks of Teeth.

No. of Teeth.	p' = Pitch in Inches. 1	1¼	1½	1¾	2	2¼	2½	3
13	129	160	193	225	257	289	321	386
14	69	87	104	121	139	156	173	208
15	49	62	74	86	99	111	123	148
16	40	50	59	69	79	89	99	191
17	34	42	50	59	67	75	84	101
18	30	37	45	52	59	67	74	89
20	25	31	37	43	49	56	62	74
22	22	27	33	39	43	49	54	65
24	20	25	30	35	40	45	49	59
26	18	23	27	32	37	41	46	55
30	17	21	25	29	33	37	41	49
40	15	18	21	25	28	32	35	42
60	13	15	19	22	25	28	31	37
80	12		17	20	23	26	29	35
100	11	14			22	25	28	34
150		13	16	19	21	24	27	32
Rack.	10	12	15	17	20	22	25	30

Centers for Faces of Teeth.

No. of Teeth.	1	1¼	1½	1¾	2	2¼	2½	3
12	5	6	7	9	10	11	12	15
15		7	8	10	11	12	14	17
20	6	8	9	11	12	14	15	18
30	7	9	10	12	14	16	18	21
40	8		11	13	15	17	19	23
60		10	12	14	16	18	20	25
80	9	11	13	15	17	19	21	26
100					18	20	22	
150			14	16	19	21	23	27
Rack.	10	12	15	17	20	22	25	30

THE WILLIS ODONTOGRAPH.



For a rack the pitch line DE becomes a straight line, and DA and EA are perpendicular to it at a distance equal to the pitch. Numbers for pitches not inserted in table may be obtained by direct proportion from the column of some other pitch; thus for 4″ pitch by doubling those of 2″. No numbers are given for 12 teeth in the upper table because for this number of teeth the profiles of the teeth within the pitch circle are radial lines,

that is $$r_0 = .955p' = \frac{3}{p}.$$

Assume 18″ wheel No. 1 diametral pitch. Find corresponding circular pitch and then by interpolation the suitable tabular numbers.

There is a circular approximation to the teeth profiles arranged by Mr. George B. Grant which possesses a high degree of accuracy. Arcs are passed through three points of face and flank, namely, through the extreme and the half-way points. A table gives the location of the centers of these arcs and their radii. See Grant's "Hand-book on the Teeth of Gears" (2d Edition), or his article, "A New Odontograph," in *Journal Franklin Institute* for February, 1887.

PROFILING OF TEETH BY RECTANGULAR COORDINATES.

This method is at once the simplest and most exact, involving only capacity to use the Square and to lay off distances accurately. By the addition of a table for locating reference lines, no describing, pitch or any other circle, need be drawn, thus doing away with all the preliminary draughting-room work and its possibilities of error.

In each table of coordinates, the *ordinates* are either radial or at right angles to pitch line, and the abscissas are at right angles to the ordinates. For ease of computation and convenience in laying out the profiles, the ordinates are expressed in simple decimal fractions and are arranged in several different groups of equidistant values.

In both sets the error is less than one unit in the last (that is, the fifth) decimal place; this means that with wheels less than 100 inches in diameter, the error in any coordinate will be less than 0.001 inch. Moreover the Epi-and Hypo-cycloidal table are so extensive, that even without interpolation, by simply taking the nearest number of teeth in the table, the error in any coordinate will not exceed 0.01 inch when the wheels are less than 100 inches in diameter.

The tables have been arranged with special reference to the interchangeable systems commonly used in American and English practice, but inspection will show that they can easily be used with any system and to solve any special problem. The grouping of the equidistant ordinates at the head of each table will enable the designer to readily choose the coordinates for few or many points according to the degree of accuracy desired The tables also indicate angles of tooth thrust.

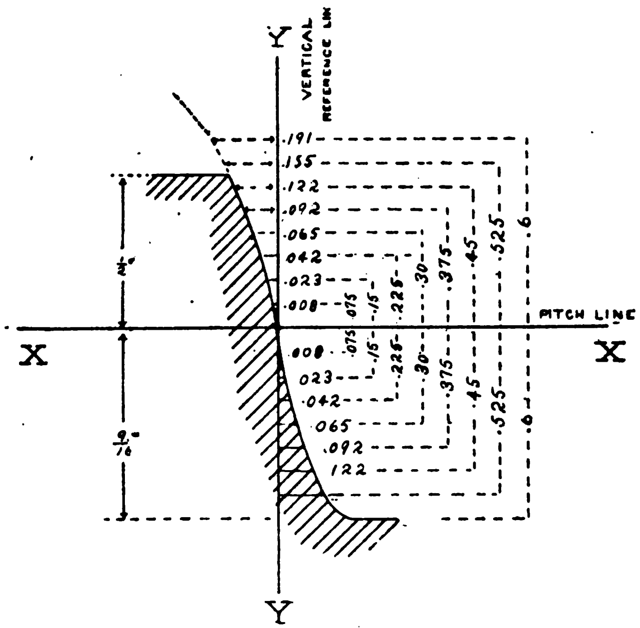
LAYING OUT OF RACK TEETH.

For these profiles the coordinate axes pass through the pitch point.

As *Involute* profiles are straight lines, the ratio of abscissa to ordinate is constant in any particular system. This ratio for different systems (*i. e.*, for different angles of tooth thrust) is given in last column of Table VII.

Example 1. Rack teeth in common system (thrust angle 75°) have No. 3 pitch. Required, coordinates of *tip* of tooth.

As the ordinary addendum (or ordinate) is $\frac{1}{3}$ inch, the abscissa of tip will be $0.2679 \times \frac{1}{3} = 0.089$ inch. The right line through this tip and the pitch point will be the desired profile.



The *cycloidal* profiles for racks belonging to the common, interchangeable, system are the same for both faces and flanks.

Example 2 (see Fig.). Suppose that for such a case the diametral pitch is 2 and that eight points on each curve are desired; as the pitch point is one of these and is at the origin, we have only to find 7 pairs of coordinates. Inspection of Table I shows us

that the middle group of equidistant ordinates gives us the desired number; the corresponding tabular values are:

	1	2	3	4	5	6	7
ordinates,	.15	.30	.45	.60	.75	.90	1.05
abscissas,	.01593	.04542	.08409	.13050	.18389	.24377	.30983

Dividing these by the diametral pitch 2 we get the coordinates used and inscribed in figure on page 51.

The exact coordinates of the tip are $\frac{1}{2}$ inch and $\frac{0.28714}{2} =$ 0.144 inch. The very same profile could have been obtained from Table III, provided we had multiplied its tabular values by hree inches, the corresponding diameter of describing circle.

If the teeth were given in terms of circumferential pitch, Table II is the one that should be used for ready computation.

Example 3 (see figure p. 53.) Teeth have No. 2 pitch. Face and flank are unlike, the former being generated by a 4 inch describing circle and the latter by a 2 inch circle. The coordinates of 5 points on curve are to be found.

The ordinary addendum is $\frac{1}{2}$ inch, which in terms of diameter of describing circle is 0.125. Inspection of Table III shows that for the faces in this case we must use the second group of equidistant ordinates, the corresponding tabular values of the coordinates are:

	1	2	3	4	5
ordinates,	.025	.050	.075	.100	.125
abscissas,	.00266	.00758	.01401	.02175	.03065

Multiplying by 4, the diameter of the describing circle, we get the coordinates for the face in figure on page 53.

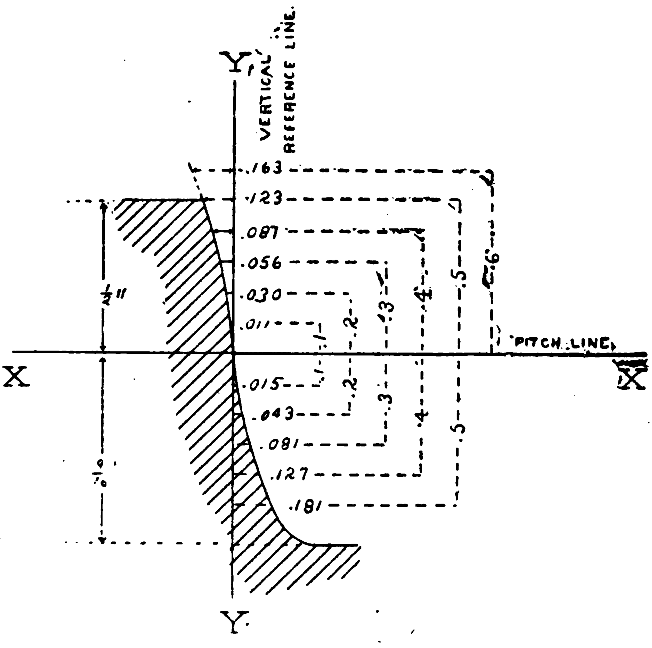
For the flanks, if we assume depth equal to ordinary addendum, we have for the ordinate of lowest point on curve (in terms of diam. describing circle), the value $0.5 \div 2 = 0.25$, and to get 5 points on curve we use first group of equidistant ordinates given in Table III; these give:

	1	2	3	4	5
ordinates,	.05	.10	.15	.20	.25
abscissas,	.00758	.02175	.04063	.06365	.09058

Multiplying by 2, the diameter of describing circle for flanks, we get values inscribed in figure on this page.

If a greater number of points had been considered desirable as many as 20 curve points could have been found in Example 2 and 10 such points in Example 3.

In both Examples 2 and 3, the profiles pass through the pitch points and are there tangent to the axis of ordinates.



LAYING OUT OF INVOLUTE TEETH.

In Table IV origin of rectangular axes is at pitch point.

In Table V origin of rectangular axes is at base circle.

Table IV is only applicable to the common, interchangeable system, in which line of tooth thrust makes an angle of 75° with line of centers; it is divided into a Part *A* giving coordinates above pitch circle and another *B* giving them below pitch circle.

As the ordinary addendum is the reciprocal of the diametral pitch or $\frac{1}{p}$, its tabular value is $\frac{1}{p} \div D = 1 \div$ No. of teeth. Each tabular value of the radial ordinate is therefore the ordinary addendum for a particular number of teeth and the latter may therefore be used to indicate the upper limit of the table for such problems as belong to this common system of involutes.

Example 4 (see figure p. 55). Find the coordinates of 14 points of the profile of an involute tooth on a 60 inch wheel with 60 teeth, 7 of the points lying above and 7 below the pitch circle.

To find the face of this tooth profile, look in first line of Part *A* of Table IV for the number 60 or the next lower number 57.1; underneath this we find the desired number of points in the fifth group of equidistant ordinates; we might have also reached this result by looking along the row of radial ordinates for the tabular value of ordinary addendum, namely $1 \div 60 = 0.016667$; as this is not directly given in the table we take the next higher value, which gives us 0.0175 for the maximum tabular ordinate, which is in the same vertical column as the number 57.1 found above; the fifth group above mentioned gives the following tabular values for the coordinates:

	1	2	3	4	5	6	7
ordinates,	.0025	.0050	.0075	.0100	.0125	.0150	.0175
abscissas,	.00070	.00144	.00224	.00309	.00397	.00490	.00588

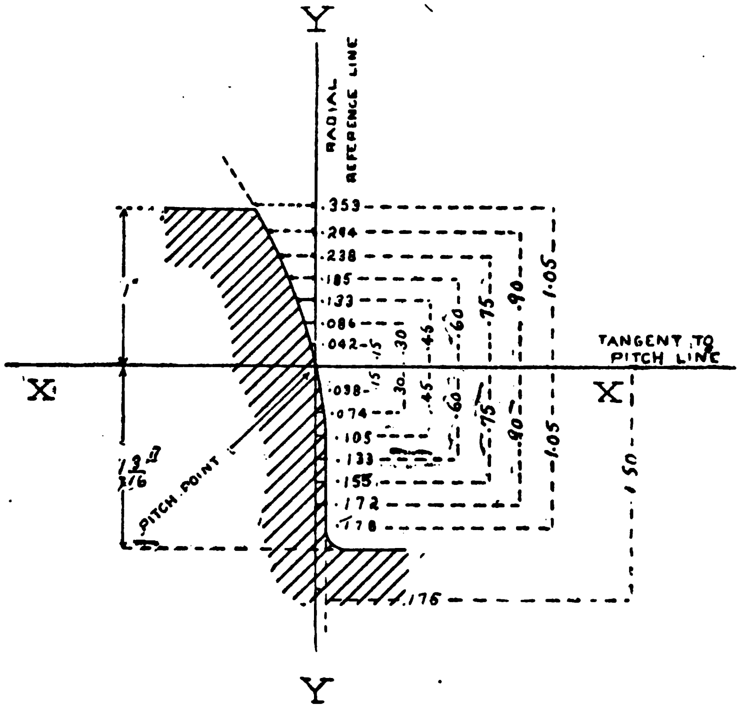
Multiplying each of these by 60, the diamerer of pitch circle, we get the coordinates inscribed in figure on page 55.

To find the *flank* or portion of profile within pitch circle, we use Part *B* of Table IV. If we take the same radial ordinates below as above the pitch line, the tooth profile will pass the base circle, there changing its character from an involute curve to a radial line and thus slightly undercutting the tooth at its root. As only the involute portion of the profile comes into action, the part below may be used to strengthen the root by making it so

that it will not interfere with the proper action of the mating tooth. The third group of equidistant ordinates gives:

	1	2	3	4	5	6	7
ordinates,	.0025	.0050	.0075	.0100	.0125	.0150	.0175
abscissas,	.00064	.00123	.00175	.00221	.00268	.00286	.00297

Multiplying by 60 we get the flank coordinates shown in Fig.



The ordinate and abscissa of the starting point of involute are respectively $.017046 \times 60 = 1.023$ and $.00297 \times 60 = 0.178$. The involute is tangent to the radius drawn through its starting point on base circle.

Thus far we have only constructed one side of tooth space. In those numerous cases in which the pitch circle is small and great accuracy is not desired, the coordinate axes for the other

side of tooth space can easily be found graphically and the coordinates laid off as before, thus obtaining the shape of the tooth space or cutter. The same may be said of the teeth of racks.

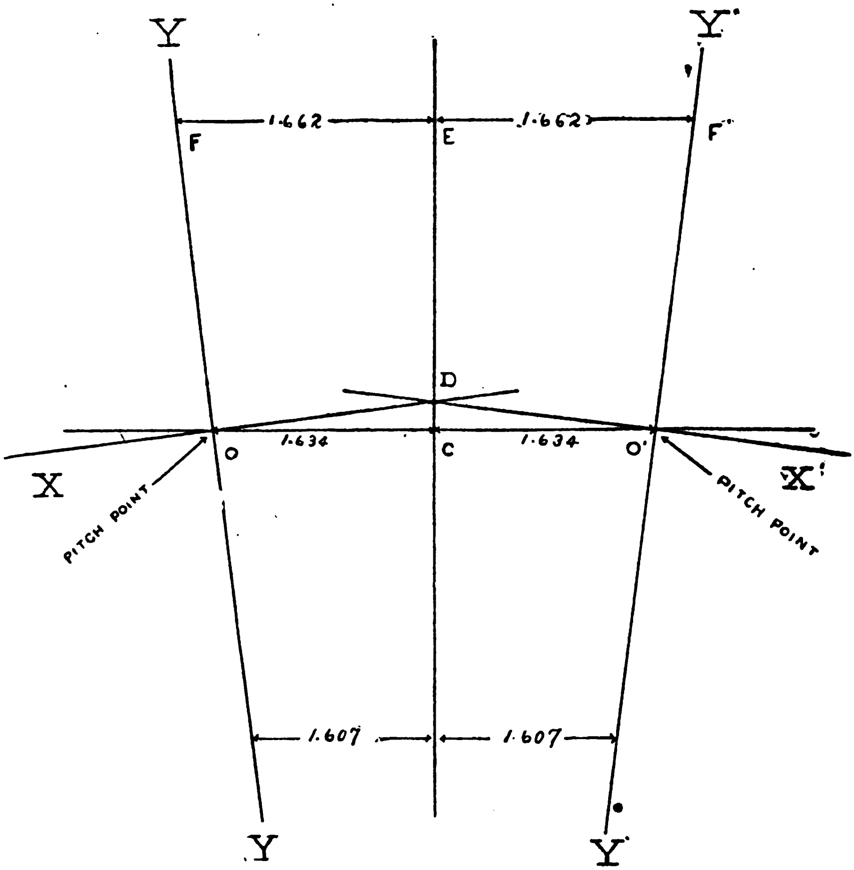
But when the wheel is a large one or great accuracy is required, it is better to locate these two pairs of coordinate axes by employing Table VI. The procedure is then as follows: Assume center line CE of tooth space and draw any line OO' at right angles to it. For a given backlash and number of teeth Table VI gives two fractions of the diameter of the wheel, which enable us to find the pitch point (or origin of the axes) and also the second point D of the tangential reference line. The radial reference line is then drawn at right angles to tangential one.

Example 5 (see figure p. 57). A 240 inch wheel has 120 teeth and 2% backlash; required the reference lines for each side of tooth space.

In Table VI opposite 120 and under .020, we find the two fractions 0.00681 and 0.00009 which multiplied by 240 give: $CO = CO' = 1.634$ inches and $CD = 0.022$ inch. The lines DO and DO' will be the required tangential reference lines, while OY and OY' at right angles to them will be the required radial reference lines. With small pitches the distance OD will be so short that there may be difficulty in drawing OY accurately perpendicular to OX. In such a case we can lay off on CE *below* C, a distance equal to $s \times CD$ and at right angles to latter a line equal to $(s + 1) \times CO$; then joining the point thus found with O we get a longer line XO on which the square may be accurately placed for erecting the perpendicular OY; thus 0.022×2 and 1.6344×3 will give respectively 0.044 and 4.903 inches, which are to be laid off *below* and to the *left* of origin C in order to get point X.

An easily applied check on the accuracy of the location of the reference lines, is to lay off 2 inches (as OF) on OY above or below O, and from F draw a line to O' Y' and perpendicular to CE.

The center line CE should exactly bisect this perpendicular and each of its halves should equal the product of $(D \pm 4) \times$ the larger of the tabular fractions; in this case $(240 \pm 4) \times 0.00681 = 1.662$ or 1.607 according as the perpendicular is drawn above or below.



Examble 6 (see figure p. 58). Data as in Example 4 and backlash $\frac{1}{2}\% = 0.005$. Required, the coordinates for laying out the tangential reference line. In Table VI under 0.005 and opposite 60, we have the fractions .01322 and .00035, which multiplied by

60 give (see Fig.) $CO = CO' = 0.793$ and $CD = 0.021$. Now joining D with O and also with O' we get the desired tangential reference lines. Or as explained above, we can lay off downwards 0.021×3 and to left of center line $0.7932 \times 4 = 3.173$ and join point X thus found with O. In either case, we draw radial reference line at right angles to tangential one.

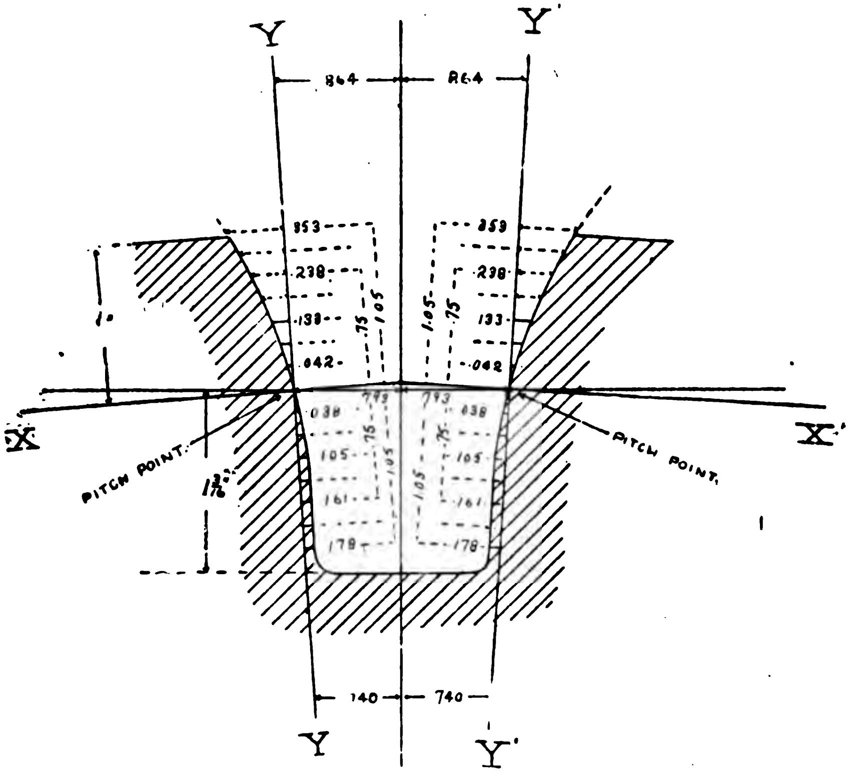
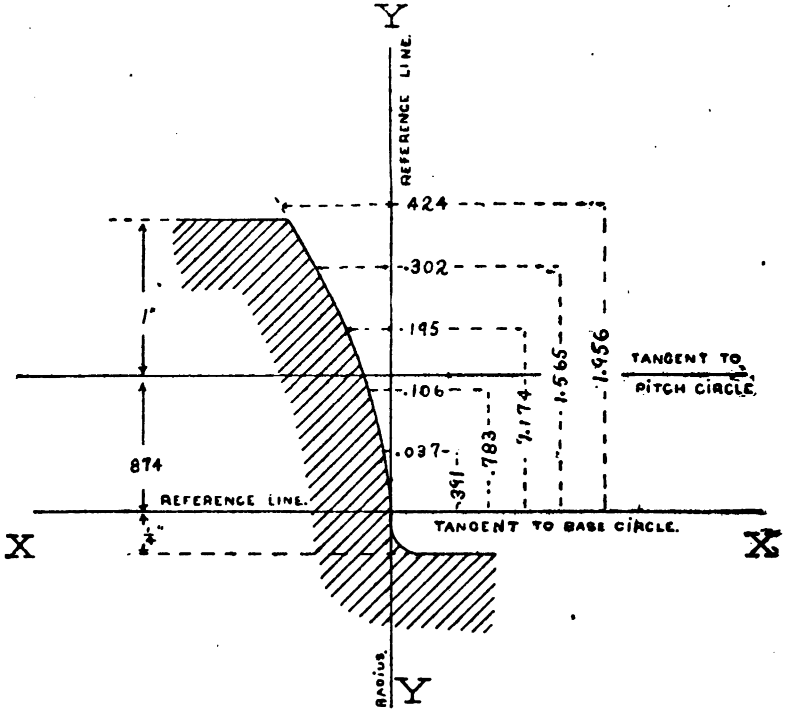


Figure is like that on p. 55, and may be regarded as the designer's free hand sketch with his computation inscribed, or as the gear maker's axact construction when laid out on a templet.

The involute profile thus far constructed belongs to the common, interchangeable system, in which tooth thrust makes an angle of 75° with line of centers. But sometimes this system is departed

from and a different angle of thrust employed. Table V is intended for such special problems, and assumes the origin of the axes on base circle, the axis of ordinates being taken radial and the axis of abscissas tangential to base circle; the tabular coordinates themselves are fractions of the diameter of the base circle. Below, *i. e.*, within the base circle, the profile is assumed to be radial.



Example 7 (see Fig.). Given an 80 inch wheel of No. 1 pitch whose tooth thrust makes an angle of 78° with line of centers; find 5 points on the profile of one side of the tooth space.

The diameter of base circle is equal to diameter of pitch circle $\times$ sin 78° (see 2d column of Table VII), or 80 $\times$ 0.97815 = 78.252. The starting point of the involute is therefore 0.874 inches inside of pitch circle and the tip is one inch outside of

this circle. The radial length of involute is therefore 1.874 inches, dividing this by 78.25, the diameter of base circle, we get the fraction 0.024; the next larger value in Table V is 0.025 and above it in the fifth group of equidistant ordinates we find 5 the desired number of points on curve; the corresponding tabular values of the ordinates are:

	1	2	3	4	5
ordinates,	.005	.010	.015	.020	.025
abscissas,	.00047	.00135	.00249	.00386	.00542

Multiplying these values by 78.25 the diameter of the base circle the products are the actual coordinates inscribed in figure on p. 59. The remainder of the profile coincides with radial reference line or axis of Y, the involute being tangent to the latter at the base circle.

To locate the reference lines for both sides of tooth space, we must in this problem make use of formulas whose terms can be found by the help of Tables VI and VII. In the former table there are under each backlash and opposite the number of teeth, two fractions, the larger of which we call M_1 and the smaller N_1. These values, it will be remembered, are directly proportional to the distances CO and CD in figure on p. 57. As in this problem the axes have their origin on the base instead of on the pitch circle, the tabular coordinates for locating origin and tangential reference line will be smaller than the similar ones given by Table VI, and we will therefore represent them by the smaller letters m and n. For these we have,

$$m = b_1 M_1 - a_1 \sqrt{0.25 - M_1^2},$$

$$n = \tfrac{1}{2}\left(\frac{1}{q} - q\right),$$

where q is the abbreviated expression for the value,

$$2a_1 M_1 + 2b_1 \sqrt{0.25 - M_1^2},$$

and a_1 and b_1 are coefficients given by Table VII for such thrust angles as would be likely to be used. Inspection of this table

shows that b_1 is so nearly unity that it may be taken as such in the great majority of problems; for instance, in computing m we can take $b_1 = 1$ for all cases in which No. of teeth > 8; also in computing n, we can take $b_1 = 1$, whenever thrust angle > 75; the error in m and n will then be less than 0.00001.

Example 8. Locate the reference lines for both sides of the space of the tooth considered in preceding example and take backlash $= 1$ %.

For 80 teeth and backlash $= .010$, Table VI gives $M_1 = 0.01001$ from which we find $\sqrt{0.25 - M_1^2} = 0.4999$; as $a_1 = .00312$ we find $m = 0.01001 - 0.00156 = 0.00845$. We now compute n by first finding $q = 0.00624 \times 0.01001 + 2 \times 0.4999 = 0.99986$, hence

$$n = \tfrac{1}{2}\left[\frac{1}{0.99986} - 0.99986\right] = 0.00014.$$

Multiplying m and n each by 78.25, the diameter of the base circle, we have 0.661 and 0.011 respectively. These two coordinates are used to locate the origin and the reference line tangential to base circle, in the same way that CO and CD (Fig. p. 57) were used to locate pitch point O and reference line DO.

Laying out of Epi- and Hypo-cycloidal Profiles.

The axes have their origin at pitch point of curve, one of them being tangential to pitch circle and the other radial. The actual coordinates are obtained by multiplying tabular values by diameter of pitch circle. As the ratio of describing circle to pitch circle is given for the whole range of values arising in practice, the two tables will be found applicable to any system in which rolling circles on circles is the means employed to generate tooth profiles. The system most used in practice has of course been specially favored in the arrangement. The procedure is like that used with Table IV and need not be explained again in detail. Attention may however be called to the asterisks in the body of the table; they mark the ordinate of the tip of the tooth belong-

ing to the *common* system or the ordinate of the very next point beyond the tip. These astericks therefore represent the limit of the table for this system, and will be found convenient in deciding upon the number of equidistant points to be used in laying out the curve. The right hand margin of table gives coordinates of points just a little beyond the tip of a *pointed* tooth, whose profiles have been generated by the common describing circle. The heavy bars between columns indicate the angle of thrust and will be serviceable in the solution of special problems.

Example 9 (Fig. p. 63). A 240 inch wheel having 120 teeth and belonging to the common system, is to be provided with epicycloidal faces and hypo-cycloidal flanks and each is to be determined by ten points.

As the pitch point is one of these, the number to be computed reduces to 9. In Table VIII opposite 120 we find an asterisk in column headed by 0.01, which is the tabular value of the maximum ordinate to be used. Above this, among the equidistant ordinates, we find that the seventh group comes nearest to what is desired, giving these values:

	1	2	3	4	5	6	7	8	10
ordinates,	.001	.002	.003	.004	.005	.006	.007	.008	.010
abscissas,	.00010	.00029	.00054	.00083	.00117	.00155	.00197	.00243	.00345

Multiplying these values by 240, the diameter of the pitch circle, we get the actual coordinates inscribed in the figure on p. 63.

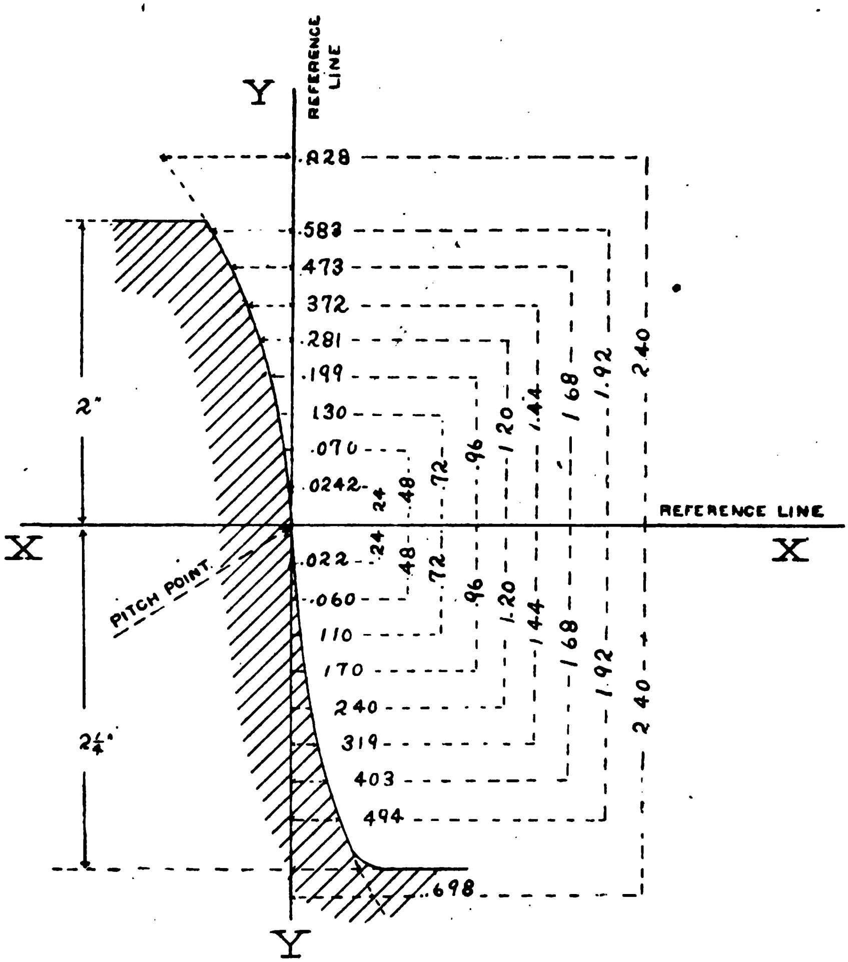
In like manner for the hypo-cycloidal flank we find in Table IX.

	1	2	2	4	5	6	7	8	10
ordinates,	.001	.002	.003	.004	.005	.006	.007	.008	.010
abscissas,	.00009	.00025	.00046	.00071	.00100	.00133	.00168	.00206	.00291

Multiplying these by 240, we get the coordinates of points of flank inscribed in figure on p. 63.

From the location of asterisks we infer that the maximum angle made by tooth thrust with line of centers is nearly 60°. The table also shows that as many as 13 points on curve could have been

found if desired, but they would not all belong to one group of equidistant ordinates. The coordinates are most numerous where curvature is most rapid, namely near the pitch point.



The face and flank are both tangent to radial axis of ordinates at this point.

The data for locating the reference lines for both sides of the tooth space are No. of teeth = 120 and backlash = 0.02; the manner of finding these lines has already been explained.

THE COMPUTATION OF THE RECTANGULAR COORDINATES.

The equations,

$$\frac{y}{r} = 1 - \cos \omega, \qquad \frac{x}{r} = w - \sin \omega,$$

give the coordinates of any point of the cycloidal profiles of a *rack*, when generating point is on the circumference of describing circle and origin of axis is on pitch line; here r is radius of describing circle and ω the angle through which latter has rolled since the generating point has left the pitch circle or centrode.

The method followed was to assume $\frac{y}{r}$ and calculate $\cos \omega$, $\sin \omega$ and ω; the second equation will give the value $\frac{x}{r}$ corresponding to assumed $\frac{y}{r}$. It is evident that here the desired values are obtained directly, that is, without trial. Before placing the results in Table III, we divide both sets by 2 so as to express them as fractions, $\frac{y}{d}, \frac{x}{d}$, of the diameter of the describing circle. For the interchangeable system in vogue in America and England, the describing circle is of such size that it will contain just 6 teeth, that is, $\pi d = 6p'$, where p' is the circumferential pitch. Moreover as $p' = \frac{\pi}{p}$, we have $pd = 6$. We can therefore express d in terms of either p' or p, the diametral pitch, and we can compute tabular values of the coordinates which will be multiples of the diametral pitch (in Table I), or fractions of the circumferential pitch (see Table II).

The coordinates y and x for the *involute* when the generating point starts from the base circle, are given by,

$$\frac{y}{R_0} = \cos \vartheta + \vartheta \sin \vartheta, \qquad \frac{x}{R_0} = \sin \vartheta - \vartheta \cos \vartheta,$$

when origin is at center of base circle and axis of ordinates is radius through involute point on base circle.

In this equation R_0 is radius of base circle and ϑ is the angle subtending the arc of the base circle rectified during the generation of the involute up to the point represented by y and x. Now it is evident from the transcendental character of these equations that x can not be expressed directly in terms of y. A tentative method was therefore pursued, the essential features of which are as follows:

A series of values, 1°, 2°, 3°, etc., was assumed for ϑ, and the corresponding values of $\frac{y}{R_0}$ were calculated from the above formula. A curve was then accurately constructed on millimeter section paper from ϑ° and $\frac{y}{R_0}$ as coordinates. Then for any assumed value of $\frac{y}{R_0}$ the corresponding value of ϑ could be accurately found from this curve. Substituting these values of ϑ in the formula for the abscissa, the values of $\frac{x}{R_0}$ corresponding to the assumed values of $\frac{y}{R_0}$ were obtained. To check the accuracy of these graphical determinations of ϑ, they were substituted in the formula for $\frac{y}{R_0}$ and the values thus obtained compared with the originally assumed values of $\frac{y}{R_0}$. If they $\left(\frac{y}{R_0}\right)$ were found to agree down to the fifth decimal place, these values of ϑ were employed in the computation of $\frac{x}{R_0}$. Before tabulating the results, the origin was transferred from center to circumference of base circle, the axis of ordinates remaining unchanged. The values of $\frac{y}{R_0}$ were changed to correspond and then both $\frac{y}{R_0}$ and $\frac{x}{R_0}$ were divided by 2, thus becoming $\frac{y}{D_0}$ and $\frac{x}{D_0}$, whereupon they were placed in Table V.

As the coordinates thus found are entirely independent of the pitch circle, they are applicable to involute profiles making any angle of thrust with the line of centers.

As the diameter of the base circle is usually a long decimal, it is not convenient for the multiplication of the tabular values. It was, therefore, thought desirable to calculate a special table for the common system of involutes which should have the coordinates expressed as decimal fractions of the diameter of the pitch circle and be referred to rectangular axes having their origin at the point where the involute cuts the pitch circle. The new axis of ordinates will then make an angle β with the former one and will pass through the pitch point of involute. Transforming the coordinates of the above equation from the old to the new set of axes, we get for the new coordinates:

$$\frac{y' + R}{R_0} = \cos(\vartheta - \beta) + \vartheta \sin(\vartheta - \beta).$$

$$\frac{x'}{R_0} = \sin(\vartheta - \beta) - \vartheta \cos(\vartheta - \beta).$$

where R is the radius of the pitch circle. Multiplying both members of each equation by $\frac{R_0}{R}$ and transposing we get:

$$\frac{y'}{R} = \frac{R_0}{R}[\cos(\vartheta - \beta) + \vartheta \sin(\vartheta - \beta)] - 1,$$

$$\frac{x'}{R} = \frac{R_0}{R}[\sin(\vartheta - \beta) - \vartheta \cos(\vartheta - \beta)].$$

To obtain numerical values we assume $(\vartheta - \beta)^\circ$ successively equal to 1°, 2°, 3°, etc., take $\frac{R_0}{R} = \sin 75^\circ$ and then calculate values of $\frac{y'}{R}$. The angle β included between the radii passing through the starting point and the pitch point can be found from the thrust angle a by combining the equations $\vartheta = \cot a$ and $\vartheta - \beta = 0.5\pi - a$, or $\beta = a + \cot a - 1.570796$; for $a = 75^\circ$ we have $\beta = 21' \, 8\frac{1}{2}''$. Plotting the assumed values $\vartheta - \beta$ and the computed values of $\frac{y'}{R}$ and drawing a curve

through the points thus found, we get the means of ascertaining the values of $\delta - \beta$ (and also of δ) for any desired values of $\frac{y'}{R}$. From these values of $\delta - \beta$ we can compute the values of $\frac{x'}{R}$ corresponding to the assumed values of $\frac{y'}{R}$. We may as before check the values of $\delta - \beta$ by computing $\frac{y'}{R}$ from them and see how perfectly they agree with the assumed values. The coordinates $\frac{y'}{R}$ and $\frac{x'}{R}$ are next divided by 2 to make them fractions of the diameter of wheel and are then arranged in their Table IV.

The coordinates of the *epicycloid*, when origin is at center of pitch circle and axis of ordinates is a radius through pitch point, are given by the equations:

$$\frac{y}{R} = \left(1 + \frac{r}{R}\right) \cos \frac{r}{R} \omega - \frac{r}{R} \cos \left(1 + \frac{r}{R}\right) \omega;$$

$$\frac{x}{R} = \left(1 + \frac{r}{R}\right) \sin \frac{r}{R} \omega - \frac{r}{R} \sin \left(1 + \frac{r}{R}\right) \omega,$$

provided the point generating the curve is in the circumference of the describing circle. Here R is radius of the pitch circle, r radius of describing circle and ω the angle through which latter circle has rolled since generating point has left centrode or pitch circle.

The transcendental character of the relation between y and x compels us to make use of tentative, graphical methods of solution. We assume $\omega^\circ = 5^\circ, 10^\circ, 15^\circ$, etc., and compute $\frac{y}{R}$ for this series. Plotting them, a curve is obtained which gives ω° for assumed values of $\frac{y}{R}$ and these values of ω in turn enable us to compute from the formula the value of $\frac{x}{R}$ corresponding to each assumed value of $\frac{y}{R}$. We check as before, transfer the origin to pitch point and divide by 2 before placing coordinates in Table VIII.

The *hypocycloidal* curve used for the flanks is given by the equations:

$$\frac{y}{R} = \left(1 - \frac{r}{R}\right) \cos \frac{r}{R} \omega + \frac{r}{R} \cos \left(1 - \frac{r}{R}\right) \omega.$$

$$\frac{x}{R} = \left(1 - \frac{r}{R}\right) \sin \frac{r}{R} \omega - \frac{r}{R} \sin \left(1 - \frac{r}{R}\right) \omega,$$

provided, the origin of the rectangular axes is at center of pitch circle, the axis of ordinates passes through the pitch point and the generating point is in the circumference of the rolling circle. The determination of the tabular values by tentative, graphical method, is exactly like that already detailed for the epicycloid. Both there and here when checking, corrections are made by the aid of the differential coefficients $\frac{dy}{d\omega}$.

THD FORMULAS USED TO LOCATE REFERENCE LINES.

It can easily be shown that the arc of pitch circle included between center line of tooth space and pitch point of tooth profile is equal to $\frac{1+c}{4}p'$, when c is the fraction of backlash and p' the circumferential pitch. Reduced to degrees and expressed in terms of the number of teeth z, the subtended angle is given by $\theta^\circ = 90\left(\frac{1+c}{z}\right)$. The $\frac{\sin \theta}{2}$ multiplied by the diameter of pitch circle gives the distance CC of figure on p. 57. It is easy to see that $\frac{\sin \theta \tan \theta}{2}$ multiplied by diameter of wheel will give distance CD of same figure. The two values $\frac{\sin \theta \tan \theta}{2}$ and $\frac{\sin \theta}{2}$ are the two fractions N_1 and M_1 accompanying each pair of arguments in Table VI.

When, as in Table V, the origin of the axis is no longer at the pitch point but on the base circle, namely, at starting point

of involute, the angle included between axis of ordinates and center line of space will be $y = \theta - \beta$. Then $\frac{\sin y}{2}$ multiplied by diameter of base circle will be distance of this new origin from the center line of tooth space; then calling $m = \frac{\sin y}{2} = \frac{1}{2} \sin (\theta - \beta) = \frac{1}{2} [\sin \theta \cos \beta - \beta \cos \theta]$ and remembering that $\frac{\sin \theta}{2} = M_1$ and $\cos \theta = \sqrt{1 - \sin^2 \theta} = 2 \sqrt{0.25 - M_1^2}$ we get by expanding and substituting,

$$m = \cos \beta \times M_1 - \sin \beta \sqrt{0.25 - M_1^2} = b_1 M_1 - a_1 \sqrt{0.25 - M_1^2}.$$

The distance that is analogous to CD in figure on p. 57 is here evidently found by multiplying $\frac{\sin (\theta - \beta) \tan (\theta - \beta)}{2}$ by diameter of base circle. Now putting

$$n = \frac{\sin (\theta - \beta) \tan (\theta - \beta)}{2} = \frac{1}{2} \left[\frac{1}{\cos (\theta - \beta)} - \cos (\theta - \beta) \right]$$

and substituting $\cos (\theta - \beta) = \cos \theta \cos \beta + \sin \theta \sin \beta =$

$$2b_1 \sqrt{0.25 - M_1^2} + 2a_1 M_1 = q, \text{ we get } n = \frac{1}{2} \left[\frac{1}{q} - q \right].$$

Cast in this form the computation is readily made, M_1 being found from Table VI and a_1 and b_1 from Table VII; in most cases $b_1 = 1$ can be assumed.

Advantages and Disadvantages of Involute and Cycloidal Forms of Teeth.

Under cycloidal forms are included the Epi- and Hypo-cycloidal varieties.

(1.) In wheels employing the cycloidal forms, the distance apart of the shafts must be exactly equal to the sum of the radii of the pitch circles of the pair of wheels which work together; even if careful construction could satisfy this condition, it is evident that wear in the bearings of the shaft would soon change this result to an imperfect one. On the other hand, wheels with

involute teeth will work correctly even when the distance between the shafts varies; for the straight line forming one of the three secondary centrodes will still touch the other two, cutting the line of centers at the pitch point (*i. e.*, point of contact of the primary centrodes), thus maintaining the velocity ratio constant. This property of involute teeth is of great importance and should be sufficient to give it the preference over the cycloidal forms of teeth.

(2.) In cycloidal forms considerable side clearance or backlash must be given in order that inaccuracies (due to the pattern, the draught, the rapping and unequal expansion) may not cause a wedging or sticking of a tooth in the opposite space into which it must enter. But where involute teeth are cast, the side clearance or backlash may be reduced to the minimum amount by simply setting the shafts nearer each other.

(3.) In cycloidal teeth the pressure on the shafts of the wheels, arising from the obliquity of the action of the teeth (*i. e.*, from the obliquity of the normals to the line of centers) is, on an average, a little less than for involute teeth; this pressure for the latter being 1.04 P(P = resistance at pitch circle), while in the former (for the ordinary addendum $= .3p'$) it is probably on an average about 1.02 P. This slight advantage is, however, more than counterbalanced by the wear and irregularity attending the continual variation in the direction and amount of the driving effort connected with the cycloidal forms of teeth.

(4.) The cycloidal shapes are not so variable as regards strength as the involute forms, for the profiles of the latter correspond very closely with the *form of uniform strength* belonging to the kind of stress to which teeth are subjected. There is also less tendency to break off the edges at the apex of the tooth in the involute forms than in the cycloidal forms, except in those cases in which the pinion is less than $\frac{1}{6}$ of the wheel.

(5.) With the cycloidal type of tooth, wheels may have fewer teeth than with the involute type; greater velocity ratios can

therefore be transmitted by the former than by the latter. The lowest number of teeth in a pinion having epicycloidal faces and radial flanks, which will work with a larger wheel having similar teeth, is equal to 3. When the wheels which work together are equal, and have epicycloidal faces and hypocycloidal flanks, the least number of teeth in each is equal to 7, while equal wheels having involute profiles have 14 teeth each, and when the wheels are unequal the lowest number of teeth in the pinion is equal to 11 while that of its partner is 15. For interchangeable wheels having involute teeth and the constant addendum $.3p'$, the least number of teeth in the pinion equals 28; a smaller number would not work correctly with a rack, the apex of the pinion tooth cutting into the radial portion of the tooth belonging to the rack. In practice, therefore, interchangeable wheels having involute teeth and constant addendum $= .3p'$ should never be made with less than 30 teeth.

(6.) The friction of cycloidal teeth is a little less than that of involute teeth.

(7.) The $\int$ shaped form of the cycloidal teeth is somewhat more difficult to construct than the simple curve of the involute teeth.

These reasons make it evident that on the whole the preference should be given to the involute type of tooth; that this has not usually been done, except in such special cases as bevel and change wheels, is probably due to the unfounded belief that the pressure on the shafts of the wheels, due to the oblique action of teeth, is much greater in the case of involute teeth than in cycloidal ones.

PLATE VII.

GEARING.

a_1 = number of pairs of teeth which must pass the line of centers before the same pair of teeth can again come in contact.

a = thickness of arm of wheel (measured parallel to axis).

b = breadth of wheel or tooth (calculated).

b' = breadth of wheel or tooth (assumed).

b_1 = number of different teeth of wheel coming in contact with same pinion tooth.

$\frac{B}{C}$ = velocity ratio of a train of wheel work in which $B > C$.

c = taper of arm of wheel $= \frac{\text{breadth of arm at rim}}{\text{breadth of arm at nave or hub}}$.

c_1 = number of teeth on pinion that come in contact with a particular tooth on wheel.

d = diameter of pitch circle of pinion (= smaller wheel of a pair).

D = diameter of pitch circle of wheel (= larger wheel of a pair).

D_1 = diameter of pitch circle of driver.

D_2 = diameter of pitch circle of follower.

e = ratio of thickness of arm to width of arm.

e_1 = greatest common divisor of Z and z.

f = permissible working stress per □″ of material of tooth.

f' = permissible working stress per □″ of material of arm of tooth.

H = number of horse-powers transmitted by wheel.

h = width of arm when prolonged to center of wheel.

h' = width of arm at nave or hub of wheel.

eh' = width of arm at *rim* of wheel.
i = side clearance, equals backlash.
j = clearance at bottom of space.
k = coefficient in formula for determining width of arm h'.
l = total length of tooth = distance between addendum and root circle.
l_1 = addendum = length of tooth above pitch circle.
l_2 = length of tooth between pitch and root circle.
L = length of shaft or axle in feet.
m = number of axes or shafts in a train of wheel work.
$m - 1$ = number of pairs of wheels in a train of wheel work having m axes.
M_1 and M_2 = bending moments of two forces acting on shaft or axle.
M_t = twisting moment of shaft or axle.
$(M_b)_i$ = ideal bending moment or resultant of several bending moments.
$(M_t)_i$ = ideal twisting moment, or equivalent to bending and twisting moments.
N = revolutions per minute of wheel.
n = revolutions per minute of pinion.
n_1, n_2 = revolutions of driver and follower respectively.
p = diametral or diametrical pitch $= \frac{Z}{D} = \frac{z}{d}$.
p' = circular or circumferential pitch $= \frac{\pi D}{Z} = \frac{\pi d}{z}$.
P = total pressure in pounds exerted on a tooth at pitch circle.
$P \times R$ = twisting moment in inch ponnds.
q = velocity ratio of driver to follower $= \frac{n_1}{n_2} = \frac{D_2}{D_1} = \frac{Z_2}{Z_1}$.
$q' = \frac{1}{q}$ = velocity ratio of follower to driver.
R = radius of wheel, *i. e.*, radius of its pitch circle.
R_1 = radius of driver, *i. e.*, radius of its pitch circle.
R_2 = radius of follower, *i. e.*, radius of its pitch circle.

S = distance between the axes of a pair of wheels.

t = thickness of tooth measured along pitch circle.

v = velocity of a point on pitch circle of wheel expressed in feet per second.

w = thickness of metal around eye of wheel.

Z = number of teeth in wheel.

z = number of teeth in pinion.

z_0 = least number of teeth allowable in a pinion.

Z_1 = number of teeth in driver.

Z_2 = number of teeth in follower.

Z_m = number of teeth in driving wheel attached to the mth axis of a train of wheel work.

z_m = number of teeth in follower wheel attached to the mth axis of a train of wheel work.

β = thickness of feather or stiffening rib on arm of wheel.

δ = thickness of rim of wheel.

θ = number of arms in wheel.

ϑ = diameter of wheel shaft (calculated to correspond to power transmitted by wheel).

ϑ' = diameter of wheel shaft actually employed.

λ = length of arm measured from outside of hub to inside of rim.

ν = number of cutters in a set.

$\psi_1 \psi_2 \psi_m$ = respectively angular velocity of the 1st, 2d and mth axis in a train of wheel work.

ω = angle included between two bending forces acting on a shaft or axle.

The distance measured along the pitch line from the face of one tooth to the similarly situated face of the next tooth is called the *pitch*. The pitch and the number of teeth in circular wheels are regulated by the following principles :

I. In wheels which rotate continuously for one revolution or more, it is obviously necessary that *the pitch should be an aliquot part of the circumference.*

In wheels which reciprocate without performing a complete revolution this condition is not necessary. Such wheels are called sectors.

II. In order that a pair of wheels or a wheel and rack may work correctly together, it is in all cases essential that *the pitch should be the same in each.*

III. Hence, in any pair of circular wheels which work together, the numbers of teeth in a complete circumference are directly as the radii and inversely as the angular velocities.

IV. Hence, also, in any pair of circular wheels which rotate continuously for one revolution or more, the ratio of the number of teeth and its reciprocal—the angular velocity ratio—must be expressible in whole numbers.

V. Let z and Z be the respective numbers of teeth in a pair of wheels, Z being the greater. Let g and G be two teeth on the smaller and larger wheels respectively, which at a particular instant work together. It is required to find, *first*, how many pairs of teeth must pass the line of contact of the pitch surfaces before g and G again come in contact (let this number be called a_1); *secondly*, with how many different teeth of the larger wheel will the tooth g come into contact (let this number be called b_1); *thirdly*, with how many different teeth of the smaller wheel will the tooth G work (let this number be called c_1).

Case I. If z is a divisor of Z,

$$a_1 = Z;\ b_1 = \frac{Z}{z};\ c_1 = 1. \tag{117}$$

Case II. If the greatest common divisor of Z and z be e_1, a number less than z, so that $z = F_1 e_1$, $Z = Fe_1$ then

$$a_1 = F_1 Z = Fz = F_1 F e_1;\ b_1 = F;\ c_1 = F_1. \tag{118}$$

Case III. If Z and z be prime to each other,

$$a_1 = Zz;\ b_1 = Z_1;\ c_1 = z. \tag{119}$$

It is generally considered desirable that each given tooth of one wheel should work with as many given teeth in the other wheel as possible, the idea being to preserve the uniformity of shape of the teeth of a pair of wheels. To accomplish this the numbers of teeth in each pair of wheels which work together should be either prime to each other, or have their greatest common divisor as small as is possible consistently with the purposes of the machine.

VI. The smallest number of teeth which it is practical to give to a wheel, is regulated by the principle that in order that the communication of motion from one wheel to another may be continuous, at least *one* pair of teeth should always be in action.

A TRAIN OF WHEEL WORK consists of a series of axes, each having upon it two wheels, one of which is *driven* by a wheel on the preceding axis, while the other *drives* a wheel on the following axis. If the wheels are all in outside gearing the direction of rotation of each axis is contrary to that of the adjoining axis. In some cases a single wheel on one axis answers the purpose of both receiving motion from a wheel on a preceding axis, and giving motion to a wheel on a following axis. Such a wheel is called an *idle wheel* or *idler*, it affects the direction of rotation only, and not the velocity ratio.

Let the series of axes be distinguished by numbers 1, 2, 3, etc., . . . m; let the numbers of teeth in the *driving wheels* be denoted by Zs, each with the number of its axis affixed thus, $Z_1, Z_2, Z_3, \ldots Z_m$. Then the ratio of the angular velocity ψ_m of the mth axis to the angular velocity ψ_1 of the first axis, is the product of the $m - 1$ velocity ratios of the successive elementary combinations, *viz.*:

$$\frac{\psi_m}{\psi_1} = \frac{Z_1 \times Z_2 \times \text{etc.} \ldots \times Z_{m-1}}{z_2 \times z_3 \times \text{etc.} \ldots \times z_m};$$

that is to say the velocity ratio of the last and first axes is the ratio of the product of the numbers of teeth in the drivers to the product of the numbers of teeth in the followers; and it is obvious that so long as the same drivers and follows constitute the train, the *order* in which they succeed each other does not affect the resultant velocity ratio.

Supposing all the wheels to be in ontside gearing, then as each elementary combination reverses the direction of rotation, and as the number of elementary combinations $m - 1$ is one less than the number of axes m, it is evident that if m is odd the direction of rotation is preserved, and if even, reversed.

It is often a question of importance to determine the number of teeth in a train of wheels best suited for giving a particular velocity ratio to two axes. It was shown by *Young* that to do this with the *least total number of teeth* the velocity ratio of each elementary combination should approximate as closely as possible to 3.59. See *Willis*' Mechanism, p. 268. This would in many cases give too many axes, and as a useful, practical rule it may be laid down that from 3 to 6 ought to be the limits of the velocity ratio of an elementary combination in wheel-work.

For transmission gearing the velocity ratio is from 3 to 4, for slowly moving wheels (such as are connected with water wheels) the velocity ratio is from 5 to 6, while in hoisting apparatus operated by muscular power, the velocity ratio may rise as high as 8 and sometimes even to 12; in these last trains, however, the speed is reduced.

Let $\frac{B}{C}$ be the velocity ratio required, reduce to its least terms, and let B be greater than C.

If $\frac{B}{C}$ is not greater than 6, and C lies between the prescribed minimum number of teeth (which may be called z_0) and its double $2z_0$, then one pair of wheels will answer the purpose and B and C will themselves be the number required. Should B

and C be inconveniently large they are, if possible, to be resolved into factors, and those factors—or, if they are too small, multiples of them—used for the number of teeth. Should B or C or both be at once inconveniently large, and prime, then instead of the exact ratio $\frac{B}{C}$, some ratio approximating to that ratio and capable of resolution into convenient factors, is to be found by the method of continued fractions.

Should $\frac{B}{C}$ be greater than 6 the best number of elementary combinations, $m - 1$, will lie between,

$$\frac{\log B - \log C}{\log 6} \text{ and } \frac{\log B - \log C}{\log 3}. \qquad (121)$$

Then, if possible, B and C themselves are to be resolved each into $m - 1$ factors (counting 1 as a factor), which factors or multiples of them shall be not less than z_0 nor greater than $6z_0$; or if B and C contain inconveniently large factors, an approximate velocity ratio found by the method of continued fractions is to be substituted for $\frac{B}{C}$ as before."

The foregoing extract from *Rankine* requires some modification. In the first place, the method of continued fractions does not always give *all* the ratios of whole numbers, which are approximations to the desired ratio; sometimes, between two consecutive values given by this method, there may be found a ratio of two whole numbers, which is a closer approximation than either of the consecutive ones. The newer or more exact method is known as *Brocot's* and its theory is well stated in *Burmester's* Kinematik, pp. 486–492. A monograph published by the Society "Hütte," and entitled "Berechnung der Räderübersetzungen," contains a statement of the main features of the method, *Brocot's* tables and numerous practical applications.

In the second place, while *Rankine's* suggestions as to the number of axes are to be followed for general machinery, we may in gearing for cranes and hoists (where the speed is *reduced*, and the velocity ratio between each pair of gears may be great) approximately determine the minimum of axes as well as of teeth, by finding the minimum value of the *product* of the total number of teeth and the number of pairs of gears. In this way it is found that each pair of gears should have a velocity ratio of 9.19. See *Reuleaux*' Konstrukteur, 4th German Edition, p. 582.

Let subscript 1 refer to driver. and subscript 2 to follower and let n represent the revolution per minute, D the diameter, z number of teeth, then velocity ratio =

$$q' = \frac{n_2}{n_1} = \frac{D_1}{D_2} = \frac{Z_1}{Z_2}. \qquad (122)$$

If the velocity ratio q' and the distance S between the axes of a pair of wheels is given, we find the radii and numbers of teeth as follows:

$$R_1 = \frac{q'S}{q' \pm 1}, \qquad R_2 = \frac{S}{q' \pm 1}, \qquad (123)$$

the upper signs being employed for external gearing and the lower for internal gearing, then, by means of the circular pitch p' or the diametral pitch p, find the numbers of teeth z_1 and z_2 from the equation,

$$2\pi R = zp' = \frac{z\pi}{p}, \qquad (124)$$

taking of course the nearest integers since each wheel must contain an exact number of teeth; then from these teeth calculate the final diameters D_1 and D_2. It should be noticed that this problem cannot always be solved exactly with the ordinary pitches, that is, such as increase by aliquot parts, *viz.*, $p_1 = 1\frac{1}{8}$, $1\frac{1}{4}$, etc., or $p = 2, 3, 4$, etc.

If the distance between the axes is arbitrary the number of teeth of the wheel must be chosen in accordance with the circumstances. If it is only a question of continuous transmission of motion as in hoisting machinery driven by muscular power, we may with the ordinary proportions (given below) for the addendum and depth of tooth (l_1 and l_2) go as low as 8 with cycloidal forms, though it would be better not to go lower than 12, while with involute forms for $z = 11$, we must have $Z =$ or > 19, for $z = 13$, $Z =$ or > 17, and for $z = Z$, $z =$ or > 15. If, however, it is principally desired that the motion shall be uniform and the wear small, we must in high speeds give to the driver not less than 20 teeth, and to the driven wheel 30 to 50 (the larger number being given to the more nearly equal pairs). The larger the number of teeth in the pair the better.

The larger the number of teeth in a pair the smaller the friction, even when the teeth are perfect in shape. Now perfect forms are seldom made in practice, and still less often can they be maintained owing to wear. We consequently find that in a train of gearing the errors due to irregularity of shape in the first part of the train are increased in the latter part, and in order that the irregularity of motion may not be too great in the last wheel, it is desirable that the followers be given more teeth as the number of teeth in the train is greater; this is particularly to be observed when there is an increase of speed in the train.

With involute forms two pairs of teeth will always be simultaneously in contact when the pair consists of wheels having at least either 30 and 80, or 40 and 60, or 50 and 50 teeth. By assuming the number of teeth z_2 in one wheel the number on the other may be found from the velocity ratio q',

$$\text{thus} \qquad z_1 = q' \times z_2. \qquad (125)$$

The relation between diametral and circular pitch is expressed by:

$$p \times p' = \pi, \tag{126}$$

from this formula the following table was obtained:

Table for converting Diametral Pitch into Circular Pitch.						Table for converting Circular Pitch into Diametral Pitch.					
p	p'	p	p'	p	p'	p'	p	p'	p	p'	p
$\frac{1}{2}$	6.2831	$2\frac{3}{4}$	1.1424	12	.2618	6	.5236	$1\frac{5}{16}$	2.3936	$\frac{5}{8}$	5.0266
$\frac{5}{8}$	5.0265	3	1.0472	14	.2244	5	.6283	$1\frac{1}{4}$	2.5133	$\frac{9}{16}$	5.5851
$\frac{3}{4}$	4.1887	$3\frac{1}{2}$	.8975	16	.1963	4	.7854	$1\frac{3}{16}$	2.6456	$\frac{1}{2}$	6.2832
$\frac{7}{8}$	3.5904	4	.7854	18	.1745	$3\frac{1}{2}$	.8975	$1\frac{1}{8}$	2.7925	$\frac{7}{16}$	7.1808
1	3.1417	5	.6283	20	.1571	3	1.0472	$1\frac{1}{16}$	2.9568	$\frac{3}{8}$	8.3776
$1\frac{1}{4}$	2.5133	6	.5236	22	.1428	$2\frac{1}{2}$	1.2566	1	3.1416	$\frac{5}{16}$	10.0531
$1\frac{1}{2}$	2.0944	7	.4489	24	.1309	2	1.5708	$\frac{15}{16}$	3.3510	$\frac{1}{4}$	12.5664
$1\frac{3}{4}$	1.7952	8	.3926	26	.1208	$1\frac{3}{4}$	1.7952	$\frac{7}{8}$	3.5904	$\frac{3}{16}$	16.7552
2	1.5708	9	.3490	28	.1122	$1\frac{1}{2}$	2.0944	$\frac{13}{16}$	3.8666	$\frac{1}{8}$	25.1327
$2\frac{1}{4}$	1.3962	10	.3142	30	.1047	$1\frac{7}{16}$	2.1854	$\frac{3}{4}$	4.1888	$\frac{1}{16}$	50.2655
$2\frac{1}{2}$	1.2566	11	.2855	32	.0982	$1\frac{3}{8}$	2.2848	$\frac{11}{16}$	4.5696		

A relation between twisting moment PR or Pr and the corresponding horse-power H and speed (revolution per minute N or n) is given by

$$PR = 63025\,\frac{H}{N}\left(\text{or } Pr = 63025\,\frac{H}{N}\right); \tag{127}$$

this will be much used hereafter for transformation purposes.

Fig. 1.—*Wheel and Teeth of Cast Iron.*

The data for this problem are given in Plate VII.

If we assume tooth with thickness at root equal to half the pitch and total length (face + flank) equal to $0.7p'$, the requirements of sufficient *strength* will be met by the formula,

$$bp' = 16.8\frac{P}{f}. \tag{128}$$

This can be readily transformed into the three following equations.

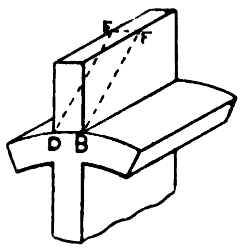
$$p' = 4.1\sqrt{\frac{P}{f}\left(\frac{p'}{b}\right)} = 1455.\sqrt{\frac{H}{fnd}\left(\frac{p'}{b}\right)} = 188.\sqrt{\frac{H}{fnz}\left(\frac{p}{b}\right)}. \tag{129}$$

As shocks are greater as the speed increases, the permissible working stress f must be taken smaller for the higher speeds. *Reuleaux* recommends for cast iron:

$$f = \frac{160800}{v + 36} = \frac{4467}{1 + 0.0001212dn}; \qquad (130)$$

for steel f is to be taken $3\frac{1}{3}$ times, and for wood $\frac{6}{10}$ times, the value for cast iron. When velocity of teeth is slow, say $=$ or $<$ 100 feet per minute, the permissible working stress f for cast iron may be regarded as invariable and equal to 4200 pounds per square inch.

When the axes of the wheels are not exactly parallel, or some foreign substance accidentally comes between the teeth, there may occur a breakage along the line *DBFE* of the accompanying figure. In such a case the maximum working stress will not exceed $1\frac{1}{2}$ times (see Weisbach-Herrman's Machinery of Transmission, p. 344) that assumed in above formulas for b and p'. With the factor of safety assumed in these formulas this stress will not cause the material to reach its limit of elasticity, but will still leave considerable margin for safety. This unusual case need, therefore, not be provided for by increasing the pitch; this is the more allowable, because smallness of pitch (by increasing the number of teeth) is favorable to smoothness of running.



The formulas thus far given do not allow for wear; following *Reuleaux* in this matter also, we take,

$$A = \frac{Pn}{b} \leqq 28000; \text{ or } b \geqq 0.0000357Pn; \text{ or } b \geqq 4\tfrac{1}{2}\frac{H}{d}. \qquad (131)$$

Here A may be regarded as a rough measure or function of the wear, the symbols n and d both belonging to the pinion, *i, e.,* to the gear teeth having the greatest number of contacts in a

given time. Deviation from this rule will not affect the strength, but will affect the rapidity of wear. So far as circumstances will permit, it is advisable to take A smaller than assumed above; it may be as low as 5600, without making the dimensions of inconvenient size. On the other hand values of $A = \frac{Pn}{b} = 67000$ sometimes occur in practice, but then they are accompanied by excessive wear. The only reason for choosing higher values for A than that given in (131), would be to make use of already existing patterns of gear wheels.

For *wood* $A = \frac{Pn}{b} < 28000$ and *Reuleaux* advises that it be kept down to 16000 or 22000.

If we combine equations 128, 130 and 131 so as to eliminate b we get:

$$p' = 0.0128d + \frac{105.3}{n} \tag{132}$$

As this corresponds to minimum value of b consistent with a reasonable degree of durability, p' will be the maximum and hence the number of teeth will be a minimum. But as numerous teeth are conducive to smooth running, it will generally be best to take p' smaller than given by the last formula.

The following are the proportions ordinarily met with in practice:

$$b > 1\tfrac{1}{2}p' \text{ and } < 4\tfrac{1}{2}p'.$$

When the wheels move slowly, *i. e.*, when $v < 1.5$ feet, as in hoisting apparatus worked by muscular power,

$$b = 2p' \text{ to } 2\tfrac{1}{2}p'. \tag{133}$$

When wheels move more rapidly (as in transmitting gear) and teeth are neither cut nor hand filed,

$$b = 2\tfrac{1}{2}p' \text{ to } 3\,p'. \tag{134}$$

When the wheels move more swiftly and have cut teeth,

$$b = 3p' \text{ to } 3\tfrac{1}{2}p. \tag{135}$$

When the wheels have a very uniform quick motion and small pitch (as is the case in many wood- and metal-working machines).

$$b = 3.5p' \text{ to } 4p'. \tag{136}$$

If we regard $A = \frac{Pb}{n}$ as a function of the wear and combine it with other equations we get three additional formulas:

$$p' = \frac{16.8}{n}\frac{A}{f}, \tag{137}$$

$$b = \frac{126050}{A}\frac{H}{d} = \frac{396000}{A}\frac{H}{zp'}, \tag{138}$$

$$z = \frac{396000}{(16.8)^2 A^3}\frac{f^2 n^2 H}{\left(\frac{b}{p'}\right)}. \tag{139}$$

DIMENSIONS OF TEETH BELONGING TO INTERCHANGEABLE WHEELS.

According to *Reuleaux*,

$$\text{Side clearance} = \text{backlash} = i = 0.05p'. \tag{140}$$

$$\text{Clearance at bottom of space} = j = 0.1p'. \tag{141}$$

$$l_1 = .3p'. \tag{142}$$

$$\text{consequently } t = \frac{p'}{2} - \frac{i}{2} = \frac{19}{40}p' = .475p'. \tag{143}$$

$$\text{and space} = p' - t = .525p'.$$

$$l_2 = l_1 + j = .4p'. \tag{144}$$

$$l = .7p'.$$

The proportions just given are suitable when the teeth are to be cast.

When the teeth are to be cut accurately we can take

$$\text{backlash} = i = 0 \left(i.\ e.,\ t = \frac{p'}{2} = \frac{1.57}{p}\right). \tag{145}$$

$$j = \frac{t}{10} = \frac{.157}{p}. \tag{146}$$

$$l_1 = \frac{1}{p}. \qquad (147)$$

$$l_2 = \frac{1.157}{p}. \qquad (148)$$

$$l = \frac{2.157}{p}.$$

The amount of backlash to be employed varies according to the accuracy of the method employed in the construction of the teeth. In cutters for forming the profiles of teeth the backlash is often made equal to 0, while teeth cast from patterns or machine molded teeth must have side clearance to allow for the inaccuracies due to the pattern, to the molding, and to the unequal contraction while cooling. But even in this case it will often be found that a backlash less than 0.05, the value given above, may be employed.

It should be remembered that even in interchangeable wheels the ratio of thickness t of tooth to pitch p' is not constant, being $t =$ or $< \frac{p'}{2}$, consequently in interchangeable wheels backlash $=$ or > 0.

Diameters of Wrought Iron Shafts.

$\vartheta =$ diameter, $P =$ pressure on tooth, $R =$ radius of wheel, $N =$ revolutions per minute, $H =$ number of horse-powers transmitted.

Formulas for strength, when $f = 6800$ pounds.

$$\vartheta = .091\sqrt[3]{PR} = 3.63\sqrt[3]{\frac{H}{N}}, \qquad (149)$$

Formulas for stiffness, when allowable twist in degrees $= \frac{3}{40}L$, is

$$\vartheta = 0.3\sqrt[4]{PR} = \sqrt[4]{\frac{H}{N}}. \qquad (150)$$

Calculated for Strength.

ϑ	PR	$\frac{H}{N}$	ϑ	PR	$\frac{H}{N}$
ins.	in. lbs.		ins.	in. lbs.	
1.2	2290	.036	4.0	84800	1.346
1.4	3040	.058	4.5	121000	1.916
1.6	5430	.086	5.0	166000	2.63
1.8	8200	.130	5.5	220000	3.50
2.0	10600	.168	6.0	286000	4.54
2.2	14100	.224	6.5	364000	5.77
2.4	18300	.291	7.0	454000	7.21
2.6	23300	.370	7.5	559000	8.87
2.8	29100	.462	8.0	678000	10.76
3.0	35800	.568	9.0	966000	15.33
3.2	43400	.689	10.0	1325000	21.02
3.4	52100	.826	11.0	1750000	27.98
3.6	61800	.981	12.0	2290000	36.33
3.8	72700	1.154	14.0	3640000	58.00
4.0	84800	1.346	16.0	5430000	86.00

Calculated for Stiffness.

ϑ	PR	$\frac{H}{N}$	ϑ	PR	$\frac{H}{N}$
ins.	in. lbs.		ins.	in. lbs.	
1.2	260	.0041	4.0	32200	.511
1.4	480	.0077	4.5	51600	.819
1.6	820	.0131	5.0	78600	1.25
1.8	1320	.021	5.5	115200	1.83
2.0	2000	.032	6.0	163500	2.60
2.2	2950	.047	6.5	225000	3.56
2.4	4200	.066	7.0	302000	4.80
2.6	5750	.091	7.5	398000	6.32
2.8	7800	.123	8.0	515000	8.18
3.0	10200	.162	9.0	826000	13.10
3.2	13200	.209	10.0	1258000	19.97
3.4	16800	.267	11.0	1842000	29.24
3.6	21100	.336			
3.8	26200	.416			
4.0	32200	.511			

The formulas and tables make no allowance for key-way. Twice the depth of the latter (or thickness of key) should be added to the value of ϑ found as above.

For Cast Iron Shafts.

Multiply PR and $\frac{H}{N}$ by 2 and find the value of ϑ corresponding to this product in the preceding table.

FOR CAST STEEL SHAFTS.

Multiply equivalent of wrought iron shaft (when proof strength of cast steel is $\frac{5}{8}$ that of wrought iron) by 0.84 for strength and by 0.88 for stiffness.

Shop shafting of considerable length should be calculated for stiffness.

The axles and shafts of ordinary machinery are usually short, the amount of twist is therefore small and may be neglected, but when such a shaft is also subjected to bending stresses, account may be taken of them in a rough way by calculating ϑ for stiffness. A more accurate method when bending forces are considerable is as follows:

Combine the bending moments of the various bending forces by means of

$$(M_b)_i = \sqrt{M_1^2 + M_2^2 + 2M_1M_2 \cos w}, \qquad (151)$$

= ideal or equivalent bending moment.
w = angle included between two forces (whether their directions intersect or not). Then ideal twisting moment

$$= (M_t)_i = 0.6\,(M_b)_i + \sqrt{(M_b)_i^2 + M_t^2}. \qquad (152)$$

M_t = actual twisting moment.

Substituting the value of $(M_t)_i$ for PR in the above formula or table we get the required ϑ.

By the method of Graphical Statics we can more quickly find $(M_t)_i$.

RIM AND ARM OF WHEEL.

$$\delta = 0.12 + .4p' = \frac{5}{4p} + \frac{1}{8}. \qquad (153)$$

According to *Reuleaux*,

$$\beta = 0.8\delta, \qquad (154)$$

$$\text{Number of arms} = \theta = 0.56\sqrt{Z}\,\sqrt[4]{p'} = 0.73\sqrt{Z}\sqrt[4]{\frac{1}{p}} \qquad (155)$$

To find a *Reuleaux* assumes value for $\frac{h}{p'}$; a good proportion generally is

$$h = 2 \text{ to } 2\tfrac{1}{2}p' \tag{156}$$

then substitute in

$$\frac{a}{b} = .07\frac{Z}{\theta}\left(\frac{p'}{h}\right)^2; \tag{157}$$

or obtain from the following table $\frac{a}{b}$ for given $\frac{h}{p'}$ and $\frac{Z}{\theta}$.

DIMENSIONS OF ARMS OF WHEELS (REULEAUX).

$\frac{h}{p'}$	Values of $\frac{a}{b}$ when								
	$\frac{Z}{\theta} = 7$	9	12	16	20	25	30	35	40
1.50	0.20	0.28	0.37	0.50	0.62	0.78	0.93	1.08	1.24
1.75	0.16	0.21	0.27	0.37	0.46	0.57	0.69	0.80	0.91
2.00	0.12	0.16	0.21	0.28	0.35	0.44	0.53	0.61	0.70
2.25	0.10	0.12	0.17	0.22	0.28	0.35	0.41	0.48	0.55
2.50	0.08	0.10	0.13	0.18	0.22	0.28	0.34	0.39	0.45
2.75	0.06	0.08	0.11	0.15	0.18	0.23	0.28	0.32	0.37
3.00	0.05	0.07	0.09	0.12	0.16	0.19	0.23	0.27	0.31

If the resulting thickness of rib a is too great or too small with respect to looks or casting, assume a new value of $\frac{h}{p'}$ and calculate over again.

DIMENSIONS OF ARMS.

When the condition is to be fulfilled that the stresses at the ends of the arm (at rim and hub) be equal, then according to *Grashof*,

if h' = breadth of arm at nave (h is measured at center of wheel),
ch' = breadth of arm at rim.
$eh' = a$ = constant thickness of arm (influence of lateral rib neglected).
λ = length of arm measured from hub to rim,

we must have

$$h' = \sqrt[3]{\frac{6P\lambda}{e(1 + c^2)\,\theta f'}}, \qquad (158)$$

Unwin gives $w = 0.4\sqrt[3]{Rp'^2} + \frac{1}{2}$, (159)

and when $c = \frac{4}{5}\ \frac{3}{4}\ \frac{7}{10},\ \frac{2}{3}$, we have

$$\frac{\lambda}{w + \frac{\vartheta}{2}} + 1 = \frac{R - .8p'}{w + \frac{\vartheta}{2}} = 2.05,\ 2.64,\ 3.7,\ 5.0, \qquad (160)$$

from which, if w and $\frac{\vartheta}{2}$ are known, the value of c may be found.

If now on account of such unknown straining actions as the unequal distribution of the load on the arms and those due to contraction while cooling, we take $f = 1000$ pounds;

also $a = eh = t = 0.48p'$, (161)

we can transform the above values of h' into

$$h' = k\sqrt{\frac{P\left(w + \frac{\vartheta}{2}\right)}{p'}}, \qquad (162)$$

k being obtained from the following table:

VALUES OF k IN PRECEDING FORMULA FOR h'.

Values of	Number of Arms of Wheel $= \theta$.				
c	$\theta = 3$	4	6	8	10	12
$\frac{4}{5}$	.052	.045	.036	.031	.028	.026
$\frac{3}{4}$	.066	.057	.047	.040	.036	.033
$\frac{7}{10}$	.087	.074	.061	.054	.047	.043
$\frac{2}{3}$	.107	.093	.076	.066	.059	.054

When $f = 2000$ or 3000 pounds the quantities in the table must be multiplied by .71 and .58 respectively.

Unwin recommends a taper of $\frac{1}{32}$ on each side for small wheels and $\frac{1}{48}$ for large wheels.

Calculate dimensions from data given on title of Plate. Full size.

Figs. 2 and 3.—*Rim and Arm Sections of Different Style from Fig. 1.*

Assume $b = 4''$, $p = 2$, $t = 0.785''$. Calculate α and β as in Fig. 1.

Equidistant Values of Cutters.

(A 12-Toothed Pinion Being Regarded as the Smallest and a Rack as the Largest Wheel in the Set.)

In making a set of cutters (especially for small pitches) it is by no means necessary to make one for every wheel, as the forms for adjacent numbers of teeth are so nearly alike that the errors of workmanship entirely destroy the difference. The variation of form is much less among high numbers than low ones. For example, the difference of form between a cutter for 150 teeth and one for 300 is not greater than that between 16 and 17 teeth. The following middle series will give a set of equidistant cutters, that is, a table of cutters so arranged that the same difference of form exists between any two consecutive numbers. The upper and lower series give the limits between which each cutter can be applied:

Numbers of Teeth for which Cutters can be employed.

Upper Limit,

$$\text{Rack}\ \frac{24v}{2}\ \frac{24v}{4}\ \frac{24v}{6}\ \ldots\ \frac{24v}{2v-4}\ \frac{24v}{2v-2}. \qquad (163)$$

Exact number,

$$\frac{24v}{1}\ \frac{24v}{3}\ \frac{24v}{5}\ \frac{24v}{7}\ \ldots\ \frac{24v}{2v-3}\ \frac{24v}{2v-1} \qquad (164)$$

Lower Limit,

$$\frac{24v}{2}\ \frac{24v}{4}\ \frac{24v}{6}\ \frac{24v}{8}\ \ldots\ \frac{24v}{2v-2}\quad 12. \qquad (165)$$

Number of Cutter,

$$1,\ 2,\ 3,\ 4,\ \ldots\ (v-1)\quad v.$$

In this series v = number of cutters in the set.

When the circular pitch is $2''$ the difference between the *bases* of two cutters which are exact for a 12-toothed pinion and rack respectively, is not more than $\frac{1}{4}''$ (the base of a cutter corresponding to distance between two adjoining teeth measured along the addendum circle); from this the difference may be ascertained for any smaller pitch, and as many cutters interposed as may seem to be necessary, the accuracy of the gear cutting machine (principally that of the index plate) being taken into account as well as the accuracy of the workmen.

Thus, if the limit of accurarcy in forming cutters be $\frac{1}{100}$th of an inch and a set of cutters is being made for $\frac{1}{2}$ pitch (*i. e.*, $p' = \frac{1}{2}$) where the maximum difference of form equals $\frac{1}{4} \times .25''$ $= .06''$, then half a dozen cutters will be sufficient and these must be made to suit as nearly as possible 144, 48, 29, 21, 16 and 13 teeth.

Problem.—Suppose the limit of accuracy is, that the distance of the apex of a tooth formed by cutter from the apex of the strictly correct tooth shall be less than $.01''$.

Find the number of cutters necessary for interchangeable wheels having $p = 2$ or $p' = 1.57$.

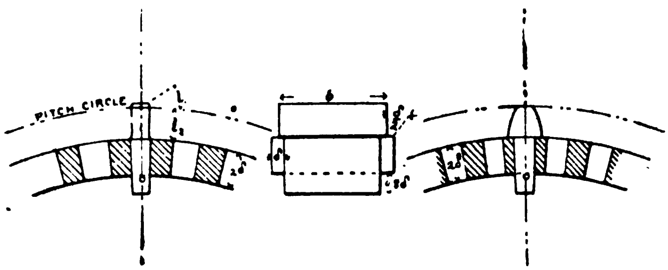
The method given above for ascertaining the cutters of equal variation is due to *Willis*. *Mr. George B. Grant* proposed the formula

$$t = \frac{an}{n - s + \frac{an}{z}}, \qquad (166)$$

in which a is the first and z the last tooth, usually 12 and ∞, of a series of n intervals. s is the number, in the series, of any particular interval, and t is the last tooth in the interval s. This formula universally distributes, not the differences in form, but the differences in lengths of the addendum arcs. *Prof. C. W. MacCord* by a "locus" or graphical method does take account of difference of form at tip of tooth; on page 194 of his "Kinematics" he gives a table comparing his own method with

that of *Willis* and of *Grant*. Absolute accuracy is not claimed for any of these methods. The most exact would probably be to make the maximum variations from uniform velocity ratio as nearly alike as possible for every pair of imperfectly shaped gears produced by the cutters. By measuring the angles at which the normals cut the pitch circles, the variation of velocity ratio could easily be computed. In locating the normals a graphical method could be used, one profile and its normals being drawn on paper and its mate on tracing cloth; contact between points would exist when their normals coincide. The increased accuracy thus obtained would not be worth all this trouble.

Figs. 6, 7 and 8.—*Wooden Teeth or Cogs* are employed with high speeds because they are better adapted



than iron ones for absorbing the shocks attending great speeds. The wooden teeth are given only to one (usually the driver) of the two wheels working together. When the larger wheel receives the wooden teeth the latter last longer on account of the less frequent contact to which they are subjected. There is a compensatory advantage, however, in giving the teeth to the smaller wheel provided a reserve pinion be kept on hand, namely, there will be no delay for repairs, as the worn out pinion can be quickly changed for the new one.

Radial flanks should be employed for wooden teeth, for then the fiber of the wood may be made parallel to the flanks, and

the iron tooth in contact with it will then not act as a tool cutting against the grain.

When the wooden tooth is given no face (the acting portion being confined to the flanks, as in Fig. 6) it should be placed on the driven wheel, as action or contact between the teeth will occur whilst they are receding from the line of centers, such action being favorable to smoothness of running

For wooden teeth take

$$f = \frac{96500}{v + 36} = \frac{2800}{1 + 0.0001212dn}, \qquad (167)$$

$$A = \frac{Pn}{b} = 20000. \qquad (168)$$

$$\text{or} \quad b = 0.00005Pn, \qquad (169)$$

$$\text{and} \qquad bp' = 16.8\frac{P}{f}. \qquad (170)$$

When practicable a smaller value can be assumed for A and the durability with respect to wear increased. By a suitable combination of these formulas expressions similar to those found for cast iron teeth can be obtained.

In Fig. 6,

$$l_1 = 0.1p', \quad l_2 = 0.6p', \quad t = 0.475p'. \qquad (171)$$

Its iron fellow has

$$l_1 = 0.5p', \quad l_2 = 0.2p', \quad t = 0.475p'. \qquad (172)$$

In Fig. 8, the profile is of involute form and has the same dimensions as its fellow.

Fig. 7 is common to both Fig. 6 and Fig. 8.

Reuleaux, p. 595, 4th German Ed. and p. 689 French Ed., gives proportions of shrouded pinions.

Unwin, p. 338, also treats of these pinions.

PLATE VIII.

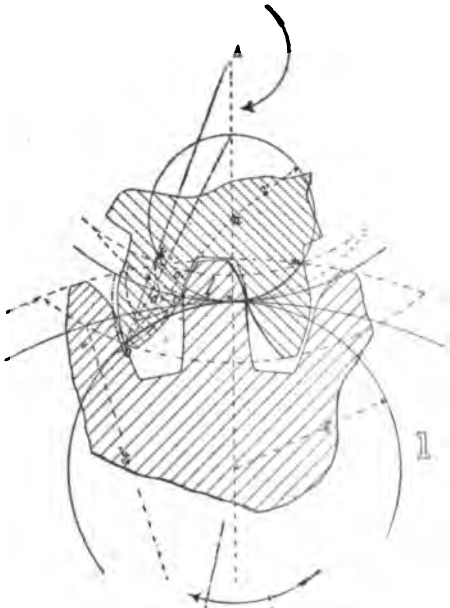
PROBLEMS IN GEARING.

THE LOCUS OF CONTACT is the line traversing the various positions of the point of contact. Since the profiles that come into contact are generated by a point on the same auxiliary or secondary centrode, the successive positions of this point will constitute the required locus of contact.

For the cycloidal or involute profiles ordinarily occurring in practice the locus of contact coincides either with the describing circles when the latter are tangent to the pitch circles at the pitch point, or with the generatrix of the involute curve when it passes through the pitch point.

That under these circumstances the positions of the generatrices will be represented by stationary lines, will be evident if we remember that in the first case (cycloidal profiles) the auxiliary centrodes roll on the primary ones in such a way as to always touch the latter at the instantaneous center (*i. e.*, point of contact of the primary centrodes) and that when the latter (the pitch point) is stationary the describing circle's center is also fixed, though its circumference moves with the circumference of the pitch circle; in like manner, in the second case (involute profiles), since the tangent to the scondary centrodes must always pass through the instantaneous center, when the latter is stationary the tangent (or generatrix of the involute) will also be fixed in *direction*, though constantly rolling on the moving base circle.

The locus of contact of the profiles is divided by the pitch point into two parts—the locus of approach described by the point of contact of the profiles in approaching the pitch point, and the locus of recess described by the point of contact after having passed the pitch point.



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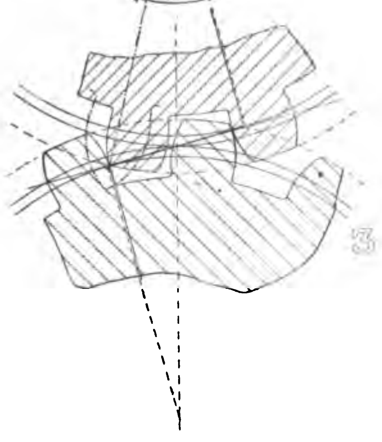

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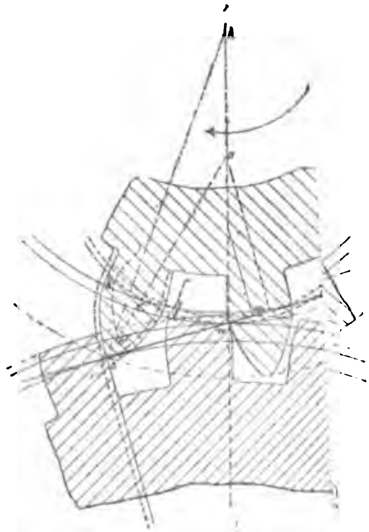
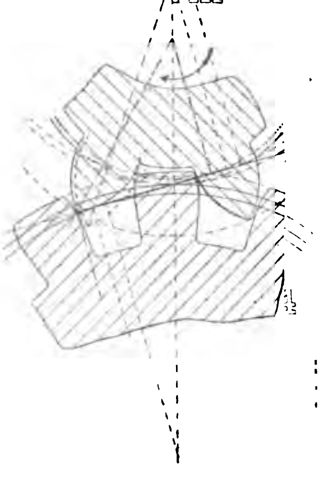


GEARING PROEM

VIII



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The locus of contact is bounded, where the approach commences, by the addendum line of the driven wheel (or follower), and where the recess terminates, by the addendum line of the driver. The length of the locus of contact must be such that there will always be one pair of teeth in contact, and it is better still, when practicable, to make it so long that there shall always be at least two pairs of teeth in contact; but this is not always possible, particularly in interchangeable wheels having a moderate and constant addendum.

THE ARC OF CONTACT ON A PITCH LINE is that part of the pitch line which passes the pitch point during the action of one given tooth on the corresponding tooth of the other wheel

In order that at least one pair of teeth may be in action at each instant, the length of the arc of contact must be *greater than tne pitch*, and when practicable it should be *double the pitch.* The two tables on pages 99 and 100 give the duration of contact in terms of pitch, for interchangeable wheels having cycloidal and involute teeth respectively. The arc of contact should never be less than $1.1p'$ and should generally be greater than $1.2p'$ It is divided by the pitch point of the tooth to which it belongs, into two parts—the *Arc of Approach* lying in advance of the front of the tooth, and the *Arc of Recess* lying behind the front of the tooth. When the wheels are to rotate in either direction, it is usual to make the arcs of approach and recess of equal length and in that case each of them must be greater than half the pitch, and should, if practicable, be made equal to the pitch. In the wheels which are to move in one direction only, the arc of recess may be made greater than that of approach, for practical experience shows that the friction during the approach is more hurtful (inducing rapid wear) than during the recess. This is due to a greater pressure on bearings during the approach than during the recess, for the line of the actual tooth thrust (resultant of normal thrust and of friction) makes a greater angle with the line of centers during the approach than during the

recess. Moreover, the lever arm of the driving force is smaller during the approach than during the recess. Shortening the arc of approach and correspondingly increasing the arc of recess (by increasing the addendum of the driver) is, therefore, an advantage whenever it can be done without too greatly increasing the obliquity of the line of action. A large value of arc of approach $= e_1$ prescribes a large addendum in the follower, and a large arc of recess $= e_2$ prescribes a large addendum in the driver. Nevertheless, making the addendum the same in each pair of wheels is not equivalent to making the arc of approach $= e_1 = e_2 =$ arc of recess, as may be seen in the following table in which unequal wheels having the same addendum are supposed to be paired.

Kind of profiles employed.	Wheel drives.	Pinion drives.
Involutes.	$e_1 < e_2$	$e_1 > e_2$
Radial flanks and epicycloidal faces on both wheel and pinion.	$e_1 > e_2$	$e_1 < e_2$
Epicycloidal faces and hypocycloidal flanks obtained by constant describing circles, as in interchangeable wheels.	$e_1 < e_2$	$e_1 > e_2$

In the last kinds of profiles $e_1 = e_2$ when the wheels are equal. Before proceeding to give the formulas and tables relating to the least number of teeth we will give the following small table (calculated by *Reuleaux*) giving the arcs of contact of pairs of wheels in which the follower has radial flanks only, and no faces while its partner element, the driver, has epicycloidal faces but no flanks; evidently in this case there will be no arc of approach, the whole action taking place behind the line of centers. This kind of tooth is sometimes employed for mortise wheel and fellow, the radial flanks being given to the wooden cogs, and the faces to iron teeth.

Arc of Contact in Terms of Circular Pitch when Addendum of Driver $= \frac{7}{12}p'$.

No. of Teeth in Follower.	No. of Teeth in Driver.		
	40	100	300
30	1.48	1.60	1.67
100	2.08	2.52	2.85
300	2.50	3.36	4.33

From what has been said it is evident that, in order that continuous motion may obtain, it is necessary that during the action of one tooth its wheel should rotate through an angle equal to that angle at the center which subtends the pitch; this evidently points to a minimum or lower limit to the number of teeth that can be given to a wheel and have continuous contact assured. This minimum number of teeth in a wheel depends not only upon the form of the teeth, but also upon the number of teeth in its partner and upon the relative magnitude of the latter.

In Trundle or Pin Gearing

we have—when thickness of tooth $= \frac{1}{2}$ pitch and angle of contact $=$ pitch angle, $Z_1 =$ number of teeth in one wheel and $Z_2 =$ number of pins in the other,—

$$\left.\begin{array}{l} \dfrac{5}{Z_1^2} + \dfrac{18}{Z_1 Z_2} + \dfrac{13}{Z_2^2} \leqq .608, \\ Z_1 = \ 3 \ \ 4 \ \ 5 \ \ 6 \ \ 7 \ \ 8 \ \ 10 \ \ 14 \ \ {48, \atop \infty} \\ Z_2 = \ {117 \atop \infty} \ \ 18 \ \ 12 \ \ 10 \ \ 9 \ \ 8 \ \ 7 \ \ 6 \ \ 5, \end{array}\right\} \text{External Gearing.} \qquad \begin{array}{r} (173) \\ \\ (174) \end{array}$$

$$\frac{5}{Z_1^2} - \frac{18}{Z_1 Z_2} + \frac{13}{Z_2^2} \leqq .608. \quad \text{Internal or Annular Gearing.} \qquad (175)$$

When each wheel of the pair has radial flanks we have—when thickness of tooth $= \frac{1}{2}$ pitch and angle of contact $=$ pitch angle,

Z_1 = number of teeth of one wheel, Z_2 = number of teeth of other—

$$\frac{1}{Z_1^2} + \frac{6}{Z_1 Z_2} + \frac{8}{Z_2^2} \overline{\overline{<}} 0.076, \quad \text{for External Gearing.} \qquad (176)$$

$$\frac{1}{Z_1^2} - \frac{6}{Z_1 Z_2} + \frac{8}{Z_2^2} \overline{\overline{<}} 0.076, \quad \text{for Internal Gearing.} \qquad (177)$$

Simple formulas like the above can not easily be obtained for the other and more common kinds of gearing, but the required results can be readily obtained in all by a graphical method explained and made use of in the problems soon to follow. Investigations of this kind are, however, of little value in rapidly running gearing, where it is a good rule not to let the number of teeth in a pinion run below 20. But for hoists, cranes and whatever slow speeds are combined with high velocity ratios the low numbered pinions determined by these investigations may be employed.

FROM REULEAUX' "CONSTRUCTIONSLEHRE," p. 361.

Table of Duration of Contact of Involute Teeth in Terms of Circular Pitch p' when axes of wheels are parallel, when addendum $l_1 = \frac{1}{3}p'$ and angle made by generatrix with line of centers $= 75^\circ$.

When $l_1 = .3p'$ or $\frac{1}{p}$ the maximum error made in using present table will be $1\frac{1}{2}\%$ of tabular quantities, the tables giving too large values for $l_1 = 03.p'$ or $\frac{1}{p}$

INVOLUTES.

For Gears Having External Contact.

Z	Z = ∞	300	200	160	120	100	80	60	50	40	30	27	24	21	19	17	15	13	11
11	1.27	1.27	1.27	1.27	1.27	1.27	1.27	1.27	1.27	1.27	1.27	1.27	1.27	1.27	1.27	1.19	1.11	1.02	0.94
13	1.39	1.39	1.39	1.39	1.39	1.39	1.39	1.39	1.39	1.39	1.39	1.39	1.39	1.39	1.36	1.28	1.19	1.11	
15	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.45	1.36	1.28		
17	1.61	1.61	1.61	1.61	1.61	1.61	1.61	1.61	1.61	1.61	1.61	1.61	1.61	1.61	1.53	1.45			
19	1.72	1.72	1.72	1.72	1.72	1.72	1.72	1.72	1.72	1.72	1.72	1.72	1.72	1 70	1.62				
21	1.82	1.82	1.82	1.82	1.82	1.82	1.82	1.82	1.82	1.82	1.82	1.82	1.82	1.79					
24	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.95	1.93	1.91						
27	2.13	2.13	2.13	2.13	2.13	2.13	2.13	2.09	2.06	2.02	1.97	1.95							
30	2.27	2.27	2.24	2.22	2.20	2.18	2.15	2.11	2.08	2.04	1.99								
41	2.38	2.32	2.30	2.28	2.25	2.23	2.20	2.16	2.13	2.10									
50	2.42	2.36	2.33	2.32	2.29	2.27	2.24	2.20	2.17										
60	2.45	2.39	2.36	2.34	2.32	2.30	2.27	2.23											
80	2.49	2.43	2.40	2.39	2.36	2.34	2.31												
100	2.52	2.46	2.43	2.41	2.39	2.37													
120	2.54	2.48	2.45	2.43	2.41														
160	2.56	2.50	2.48	2.46															
200	2.58	2.52	2.50																
300	2.61	2.55																	

For Gears Having Internal Contact (Annular Gearing).

Z	Number of Teeth in Smaller Wheel.													
	Z = 11	13	15	17	19	21	24	27	30	40	50	60	80	100
300	1.27	1.29	1.50	1.62	1.72	1.82	1.98	2.13	2.27	2.45	2.49	2.52	2.56	2.59
160	1.27	1.29	1.50	1.62	1.72	1.82	1.98	2.13	2.27	2.53	2.57	2.60		
100	1.27	1.29	1.50	1.62	1.72	1.82	1.98	2.13	2.27	2.67				
60	1.27	1.29	1.50	1.62	1.72	1.82	1.98	2.13						

FROM REULEAUX' "KONSTRUCTIONLEHRE," p. 344.

CYCLOIDAL TEETH.

For Gears Having External Contact.

Z	$Z = \infty$	300	200	160	120	100	80	60	50	40	30	27	24	21	19	17	15	13	11
11	1.46	1.45	1.45	1.45	1.44	1.44	1.44	1.43	1.42	1.42	1.40	1.40	1.39	1.39	1.38	1.37	1.36	1.35	1.34
13	1.47	1.47	1.46	1.46	1.46	1.45	1.45	1.44	1.44	1.43	1.41	1.41	1.41	1.40	1.39	1.38	1.38	1.36	
15	1.48	1.48	1.47	1.47	1.47	1.47	1.46	1.45	1.45	1.44	1.43	1.42	1.42	1.41	1.40	1.40	1.39		
17	1.49	1.49	1.48	1.48	1.48	1.47	1.47	1.46	1.46	1.45	1.44	1.43	1.43	1.42	1.41	1.41			
19	1.50	1.49	1.49	1.49	1.48	1.48	1.48	1.47	1.47	1.46	1.45	1.44	1.43	1.43	1.42				
21	1.51	1.50	1.50	1.50	1.49	1.49	1.48	1.48	1.47	1.46	1.45	1.45	1.44	1.43					
24	1.51	1.51	1.50	1.50	1.50	1.50	1.49	1.48	1.48	1.47	1.46	1.45	1.45						
27	1.52	1.51	1.51	1.51	1.51	1.50	1.50	1.49	1.49	1.48	1.47	1.46							
30	1.53	1.52	1.52	1.51	1.51	1.51	1.50	1.50	1.49	1.48	1.47								
40	1.54	1.53	1.53	1.53	1.52	1.52	1.52	1.51	1.50	1.49									
50	1.55	1.54	1.54	1.53	1.53	1.53	1.52	1.52	1.51										
60	1.55	1.54	1.54	1.54	1.54	1.53	1.53	1.52											
80	1.56	1.55	1.55	1.55	1.54	1.54	1.54												
100	1.56	1.56	1.55	1.55	1.55	1.54													
120	1.56	1.56	1.56	1.55	1.55														
160	1.57	1.56	1.56	1.56															
200	1.57	1.56	1.56																
300	1.57	1.57																	

For Gears Having Internal Contact (Annular Gearing).

Z	No. of Teeth in Small Wheel.													
	$Z = 11$	13	15	17	19	21	24	27	30	40	50	60	80	100
300	1.47	1.48	1.49	1.50	1.50	1.51	1.52	1.53	1.53	1.53	1.55	1.56	1.56	1.57
160	1.47	1.48	1.50	1.50	1.50	1.52	1.53	1.53	1.53	1.54	1.56	1.56		
100	1.48	1.49	1.50	1.51	1.51	1.53	1.53	1.54	1.54	1.55				
60	1.49	1.51	1.52	1.53	1.53	1.54	1.55	1.55	1.55					

(For Table see preceding page.)

Table of Duration of Contact of Cycloidal Teeth Expressed in terms of Circular Pitch p' when Axes of Wheels are parallel aud when addendum $l_1 = \frac{1}{3}p'$, and when radius of describing circle $= r_0 = \frac{7}{8}p' = \frac{11}{4}p$, corresponding to the case when N (the number of teeth in the smallest wheel in the set) is equal to 11.

But the table may also be employed when,

$$l_1 = .3p' \text{ or } \frac{1}{p} \qquad \text{and} \qquad r_0 = 0.955p' = \frac{3}{p}$$

(corresponding to the case when $N = 12$), for the diminished addendum l_1 and increased radius r_0 neutralize each other's influence so that the maximum error involved in using the table when $l_1 = .3p'$, $N = 12$ and $r_0 = 0.955p' = \frac{3}{p}$ does not exceed $1\frac{1}{2}\%$ of the tabular quantities.

PROBLEM 1.—*Given*, diameter of pitch circle of wheel $= 24''$; arc of approach $= \frac{5}{8}p'$; arc of recess $= 0.9p'$; No. 1 pitch; width of tooth $= 0.48p'$, and describing circles for the faces and flanks of the teeth of $4''$ and $8''$ diameters respectively. *Required*, the smallest pinion that will drive the given wheel and satisfy the above conditions.

It should be noticed that in this problem the addenda are not given—they must be found. In this, and generally in similar problems, they will be found unequal. The smallest pinion that will drive the given wheel will be the one whose greatest addendum is just sufficient to give the required arc of recess.

Since the greatest addendum of a pinion is obtained by continuing the faces of a tooth till they intersect, it is evident that the apex of the tooth thus formed must be in contact with the flanks of the given wheel at the instant when the contact between a pair of wheels ends; that is, the pinion must be large enough so that the apex of its tooth will reach to the end of the locus of contact. For example, in Fig. 1, the pinion has a right-handed rotation, and the apex of its tooth o is just ceasing to be in contact with flanks of wheel. The heavily drawn line *mno*

represents the locus of contact; *mn* the locus of contact during the approach to, and *no* the locus during the recess of the teeth from, the line of centers *An*.

In order that we may find the pitch circle of the required smallest pinion, we must proceed in the following tentative manner. We assume any circle as the first approximation to the required pitch circle; in the figure the describing circle *mn* with center at *a* is so chosen.

Since the apex of the tooth must touch at *o*, the line *ao* may be regarded as the central line of a tooth on the assumed pitch circle *mn*. On each side of the central line we now lay off the half thickness of tooth *cl* and *ck* and construct at the points *l* and *k* the epicycloids *lo′* and *ko′* prolonging them till they intersect at *o′*. Now if the circle, described from *a* as a center and *ao′* as a radius, falls short of the point *o* we know that the pitch circle assumed is too small; if the circle, however, falls beyond *o* we know that the pitch circle assumed is too large. In the present case it is too small; we must, therefore, repeat the above process with a larger pitch circle and so continue till we find a pitch circle whose tooth has its apex at *o*. If this pitch circle contains an exact number of teeth of the required pitch, the problem will be completely solved; if not (and this will generally be the case), we must take the next largest size of pitch circle that will give an exact number of teeth, and must then truncate its teeth so as to give the required arc of recess *no*.

Draw full size; find maximum inclination of line of pressures to line of centers; show arc of contact on pitch circles and locus of contact in describing circles in heavy blue lines, and the auxiliary lines employed in the tentative method above described in full red lines.

Show one tooth of pinion just parting from flank of wheel and other tooth in proper contact with face of wheel. Shade teeth on pinion and wheel coarsely, so that the construction lines employed in the tentative method may be readily seen.

PROBLEM 2.—*Given*, 24″ wheel having involute teeth with arc of approach $= e_1 = 0.2p'$, arc of recess $= p'$ and generatrix making an angle of 75° with line of centers.

Required, as before, the smallest pinion that will drive the given wheel and will satisfy the conditions.

This problem may be solved in a manner similar to the preceding one, the first pitch circle assumed having the same center as the base circle (smallest) of the pinion, the latter circle being found by laying off from the pitch point on the generatrix a distance $e_1 \sin 75°$ (e_1 representing the assumed arc of approach) and at the extremity of the line laid off erecting a perpendicular; where it intersects the line of centers will be the center of the smallest possible base circle that can be employed for the pinion under the given conditions. With this point as a center we describe the corresponding pitch circle, draw a central line and construct tooth as before, and so continue till a tooth is obtained whose apex is in contact with flank of wheel at the end of the assumed locus of contact.

Direction for drawing, same as in preceding figure.

PROBLEM 3.—Find smallest pair of *equal* wheels that will work together when arc of contact $e = 1.1p'$, involute teeth of No. 1 pitch being employed.

Lay off on generatrix on each side of pitch point the distance $\frac{e}{2} \sin 75°$, at the extremities of these lines erect perpendiculars; where these intersect the line of centers, will be the centers of the required wheels (provided the corresponding pitch circles contain an exact number of teeth of the required pitch and that the teeth are sufficiently long to make arc of approach = arc of recess $= \frac{1.1p'}{2}$).

Draw full size; show two pairs of teeth in contact, and shade sections of teeth and wheels.

PROBLEM 4.—Let the teeth be involutes of No. 1 pitch, and the total arc of contact $e = 1.1p'$.

Find the smallest pinion that will drive, and satisfy these conditions when the wheels of the pair are unequal.

In this problem the arc of approach $<$ arc of recess, the former being taken as small as possible, the limit being when the assumed base or pitch circle is just large enough to allow the apices of its teeth to touch the flanks of the opposite wheel at the end of the locus of contact. This of course implies that the method is a tentative one similar to those already described. Directions for drawing, same as in preceding problem. Indicate which one of the pair of wheels is the driver and the direction of turning.

PROBLEM 5.—Construct involute teeth for a pair of elliptical wheels, and so proportion the latter that they will make the tool-holder of a shaping machine perform its return stroke in half the time of the forward stroke. The wheels are to be exactly alike and must, therefore, contain an odd, as well as equal, number of teeth. Let symbols a, b, a' and b' have same meaning as on p. 41. Take $Z = 37$, $p' = 1''$,

$$q = \frac{\text{duration of advance}}{\text{duration of return.}} = \frac{360^\circ - 2\alpha^\circ}{2\alpha^\circ},$$

$$\text{then will } \sin\alpha = \sin\frac{360^\circ}{2(q+1)}, \tag{178}$$

and, as arranged on p. 41,

$$\frac{a}{b} = \sqrt{\frac{1 + \sin\alpha}{2\sin\alpha}} = \sqrt{K}, \tag{179}$$

$$\frac{a - b}{a + b} = \frac{\sqrt{K} - 1}{\sqrt{K} + 1} = n, \tag{180}$$

$$\text{Let } K = \frac{1 + \sin\alpha}{2\sin\alpha}, \tag{181}$$

then will

$$2a = \frac{2Zp'}{\pi\left(1 + \frac{n^2}{4} + \frac{n^4}{64} + \frac{n^6}{256}\right)} \cdot \frac{\sqrt{K}}{(\sqrt{K} + 1)} = cZp', \tag{182}$$

where c is simply a function of the ratio q;

Assuming $$\left(\frac{a'}{a} + \frac{b'}{b}\right) \div 2 = \cos 15^\circ,$$

we have

$$a' = \frac{a}{a^2 - b^2}\left(-2b^2 \cos 15^\circ + \sqrt{(a^2 - b^2)^2 + 4a^2b^2 \cos^2 15^\circ}\right) \quad (183)$$

See also equation 108, p. 41, and article in "Der Civil Ingenieur," 1875, p. 223.

PROBLEM 6.—Draw annular wheel having 100 teeth with pinion of 20 teeth No. 3 pitch.

Give the teeth the same amount of addendum as in external gearing, and give radial flanks to pinion.

Take width of tooth $0.48p'$. See *Reuleaux*' "Konstrukteur," Ger. ed., p. 528.

Draw arc of contact on each of the pitch circles, and locus of contact when pinion is driver and has right-handed rotation. Find arcs of approach and recess in terms of circular pitch.

Draw half size, the center of both pinion and wheel being taken above pitch point; section-line, etc., as in previous figures.

PROBLEM 7.—Find the number of teeth in each of a pair of wheels whose axes should as nearly as possible be $17.25''$ apart, the pair transmitting a velocity ratio $= \frac{7}{8}$ and the teeth being formed by a cutter of No. $2\frac{1}{2}$ pitch. No drawing required.

PROBLEM 8.—Find the number of teeth in each of the change wheels required in a lathe for cutting 13 threads to the inch, the leading screw containing 3 threads to the inch. A change wheel is required for the spindle or mandrel of the lathe, another for the leading screw, and two change wheels for an intermediate arbor fixed to tangent plate or radial arm; there are in all 4 change wheels and three shafts or arbors.

The change wheels belonging to the lathe begin with 20 teeth and increase by 5s up to 80 teeth and then by 10s up to 120. How could a left-handed thread, $4\frac{1}{2}$ to the inch, be cut on the same lathe?

PROBLEM 9.—*No. 5 Leading Screw on Lathe.* It is required to cut a thread $1\frac{1}{8}''$ pitch, compound gearing being employed. Show the three arbors and the change wheels upon them; also state the number of teeth on each of the change wheels.

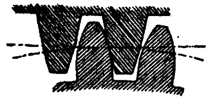
PROBLEM 10.—*No. 6 Leading Screw on Lathe.* It is required to cut a thread of one centimeter pitch (1 centimeter = 0.39370791). Find the number of change wheels and the number of teeth on each which will produce the closest approximation to the required result.

These thread-cutting problems can easily be solved by the tables contained in "Zeitschrift des Vereins deutscher Ingenieure," 1858, pp. 76 and 162.

The realization of numerically complex velocity ratios by means of toothed gearing will be greatly aided by the formulas and tables contained in the "Hütte" publication, "Berechnung der Räderübersetzungen." The method of continued fractions may also be used for this purpose, but it is more lengthy and less exact than the method of *Brocot* contained in the aforesaid publication.

BEVEL GEARING.

IX



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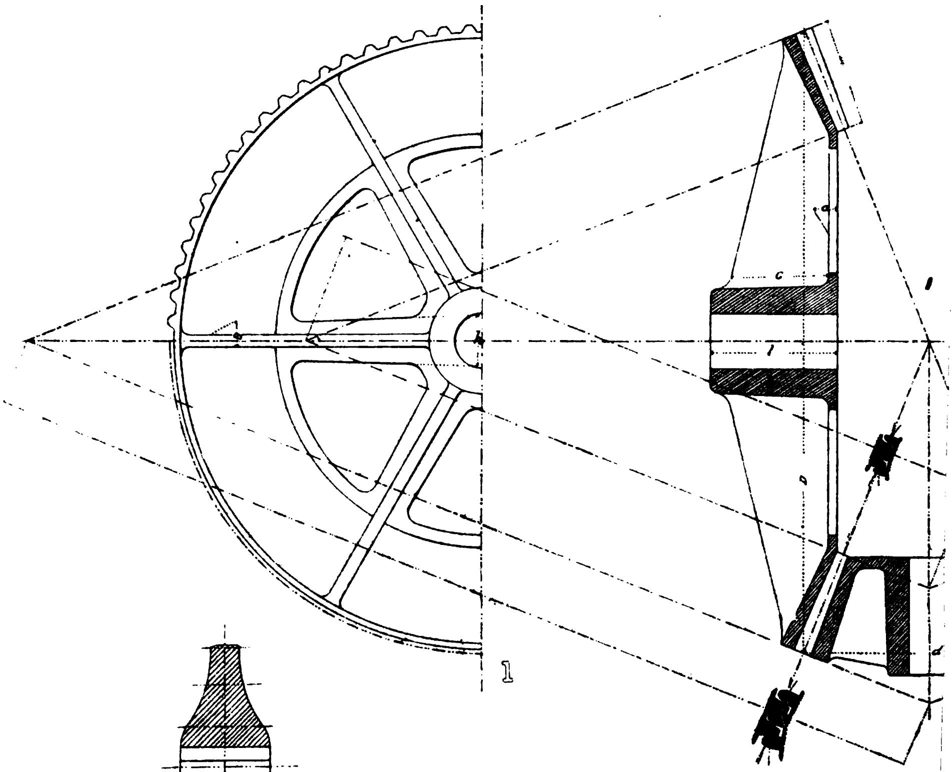

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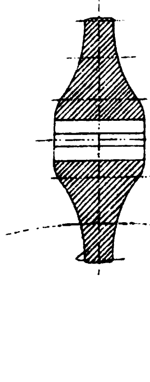

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PLATE IX.

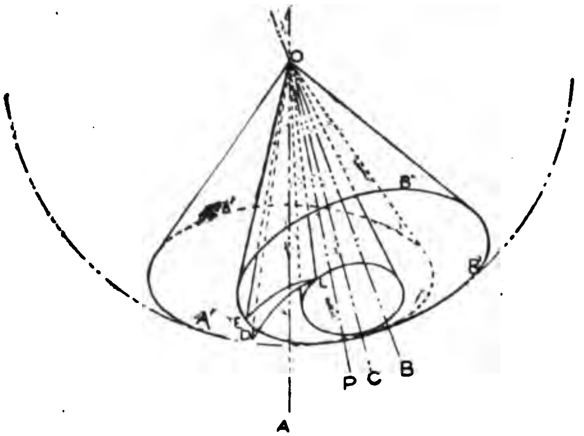
BEVEL GEARING.

b = length of tooth measured along element of pitch cone.
d = diameter of nominal (largest) pitch circle in pinion.
D = diameter of nominal (largest) pitch circle in wheel.
d_m = diameter of mean pitch circle in pinion.
D_m = diameter of mean pitch circle in wheel.
f = permissible working stress in material in pounds per □″.
H = number of horse-powers transmitted.
n = number of revolutions per minute of bevel pinion.
N = number of revolutions per minute of bevel wheel.
p' = circular pitch on nominal (largest) pitch circle.
p_0' = circular pitch on smallest pitch circle.
p = diametral pitch of normal (largest) pitch circle.
p'_m = circular pitch on mean pitch circle.
r_m = radius of mean pitch circle in pinion.
R_m = radius of mean pitch circle in wheel.
v_m = velocity in feet per second of point on middle pitch circle of wheel.
z = number of teeth in bevel pinion.
Z = number of teeth in bevel wheel.

In bevel gearing the axes of the wheels intersect. Since the point of intersection remains fixed while the wheels rotate, we have here an example of rotation about a point, the relative motion of the two wheels being represented by the *rolling* on each other of two conical axodes, whose vertices coincide with the point of intersection (see "Kinematics of Machinery," p. 76).

As in the case of the cylindrical axodes (represented by their centrodes) we may here also make use of auxiliary axodes for the

purpose of generating the surfaces of the teeth. In the ordinary kind of bevel gearing these auxiliary axodes, like their primary axodes, are right cones with circular bases. In the following figure OA and OB are the axes of the conical axodes $OA'A'$ and $OB'B'$, while OC is the axis of the auxiliary axode OPL, whose element OL generates the two conical surfaces OOL and ODL. These last two surfaces constitute respectively the flank of a tooth attached to the wheel, whose axis is OB, and the face



of another tooth attached to the bevel wheel having the axis OA. The bases DL and EL of these conical surfaces OEL and ODL were generated by a point L moving on the surface of a sphere (with radius OL) while the auxiliary axode to which it was attached was rolling on the primary axodes, EL being a spherical hypocycloid. If through any point of the instantaneous axis or element of contact OP we pass planes normal to the axis OB and OA, the intersection of these planes with the cones will be circles which roll on each other. These circles hold the same relation to bevel wheels that the ordinary pitch circle does to its spur wheel. We may indeed call them pitch circles, but in order that there may be no ambiguity as to which circle is meant it is necessary to state its distance from the vertex of the cones.

It is customary in American practice to designate the *largest* of these circles as the pitch circle.

By passing a plane through the common element of contact so as to make an angle—say of 75°—with the plane of the axes, and then rolling this plane on the right cones to which it is tangent (these tangent cones having the same axes as the primary cones) any element of the plane passing through the vertex of the cones, will generate involute surfaces whose intersection with the sphere may be called spherical involutes.

APPROXIMATELY CORRECT PROFILES FOR THE TEETH OF BEVEL WHEELS.

In order that the theoretically determined profiles of the teeth may be transferred to the actual wheels, recourse is frequently had in practice to templets, the proper form being first given to the latter and then transferred to the wooden pieces which are to form the pattern if the teeth are to be cast, or to a circular cutter if the teeth are to be cut.

But this method can not be carried out exactly (except with great difficulty) in the case of bevel wheels, for the requisite spherical cycloid or spherical involute form can not easily be given to the templets on account of the non-developable character of spherical surfaces.

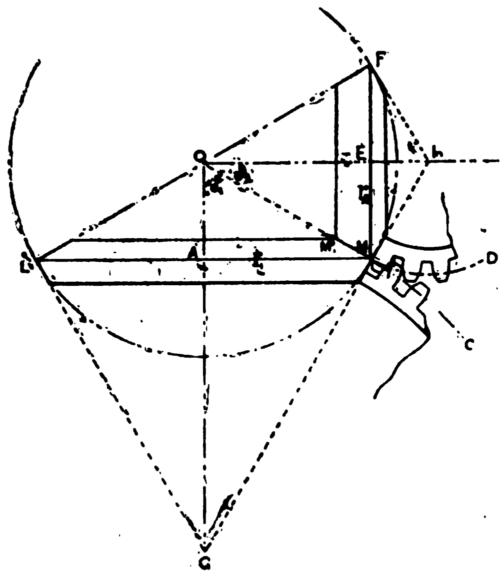
A new and exact method of planing the teeth of bevel gears has recently been introduced by *Mr. Hugo Bilgram* and his machine will be found described in *Journal Franklin Institute*, for August, 1886, p. 135. It may be regarded as a particular case of the general method suggested by *Prof. G. Herrmann* for the planing of all kinds of tooth surfaces which are capable of being generated by the motion of a straight line.

For a complete statement of the general method see *Weisbach-Herrmann's* "Machinery of Transmission," page 400. For the machine-tool modifications designed by *Prof. Herrmann* for the

planing of these gears, see "Verhandlungen des Vereins zur Beförderung des Gewerbfleisses," 1877.

Where great accuracy is not necessary the following approximation may be employed; it is one which is still resorted to extensively.

The principle of this approximate method consists in substituting for the spherical surface two right cones which are drawn tangent to that portion of the surface containing the spherical cycloids or involutes. The following figure will help to make this clearer.



OM is the radius of the spherical surface *LMF; GML* and *HMF* are the two right cones tangent to the sphere at the circles *ML* and *MF* of the bevel wheels. If we now develop the tangent cones, and treat the sectors *CMG* and *DMH*, obtained by the development, as if they were spur wheels, that is, construct the profiles of their teeth by any of the methods employed for spur wheels, the resulting profiles will serve as templets which can be

wrapped up on the tangent cones, and will then represent approximately the required spherical cycloids or involutes When the teeth are to be cast two sets of tangent cones are

$$GM = \frac{r_1}{\cos\ \delta_1}. \tag{184}$$

$$HM = \frac{r_2}{\cos\ \delta_2}. \tag{185}$$

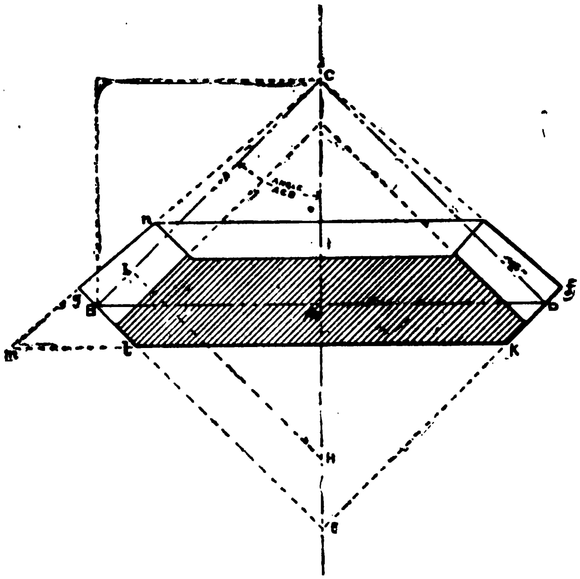
drawn, one set to the sphere containing the largest pitch circle or pitch circle proper, and the other to the smaller sphere containing the smallest or inner pitch circle. The templets thus obtained may then be fastened to the wooden pieces of the pattern from which the teeth are to be formed, the wood being cut down till the outlines of the two templets are connected by a series of rectilinear elements, all of which pass through the vertex of the cone or intersection of the axes.

When the teeth of a bevel wheel are to be formed by means of a rotary cutter, at least two cuts must be made to form one space between the teeth, for the cross section of this space varies and diminishes as it passes from the heel (*i. e.*, tangent or supplementary cone) of the wheel to the vertex.

This process involves, evidently, still another approximation, for while the side of the space or tooth is constantly varying its form, the shape of the cutter which is to produce it must necessarily remain constant, so that if the profile of the cutter is correct for one part of the tooth it will be incorrect for the other parts. Another inaccuracy attending this method of forming the tooth is that the rectilinear elements, on the side of the tooth produced by the cutter, are all parallel instead of converging to the vertex of the cone or point of intersection of the axes.

For the smaller bevel wheels these inaccuracies are not excessive, their teeth are therefore usually formed by means of cutters, the best method known to the writer being tnat devised by *Prof. C. B. Richards.*

DIMENSIONS OF BLANK.



$$gn = \frac{BC}{4} = \text{length of tooth,} \qquad (186)$$

Radii.

$$BE = BC \times \text{tang } ACB, \qquad (187)$$

$$IH = IC \times \text{tang } ACB. \qquad (188)$$

Largest Diameter of Blank.

$$fg = BD + (2Bg \times \cos ACB), \qquad (189)$$

$$BC = AC \times \text{secant } ACB. \qquad (190)$$

Angles.

$$\left.\begin{aligned} \text{tang } ACB &= \frac{AB}{AC} \\ \text{tang } BCg &= \frac{Bg}{BC} \end{aligned}\right\} \qquad (191)$$

$$Klg = 180° - ACB,$$

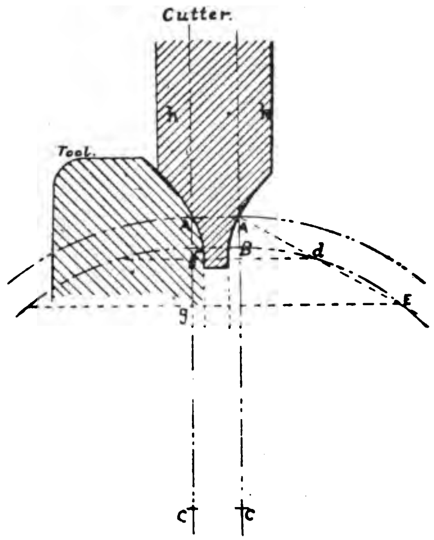
$$KmC = 90^\circ - (ACB + BCg), \qquad (192)$$

$$Bg = \frac{1}{\text{diametral pitch}} = \frac{1}{p}. \qquad (193)$$

Construction of Cutter.

AC = radius of pitch circle of cutter and corresponds to line IH of preceding figure. (194)

$BC = .966\ AC$ = radius of the circle in which lie the centers from which the arcs forming the outline of the cutter are described. (195)



$$hh = 3AA. \qquad (196)$$

$Ad = 0.15AC$ = radius of curve below pitch circle,
$Ae = 0.50AC$ = radius of curve above pitch circle,
$Af = 0.055AC$ = vertical distance of center d below A,
$Ag = 0.158AC$ = vertical distance of center e below A. (197)

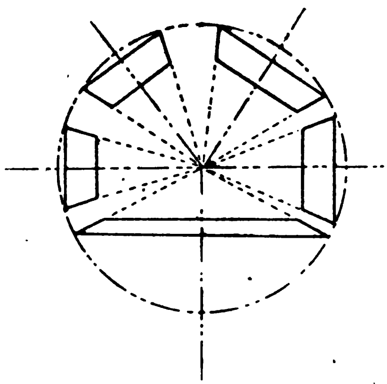
$CD = AA$ = thickness of cutter at pitch circle $= \dfrac{1}{\text{diam. pitch}} = \dfrac{1}{p}$.

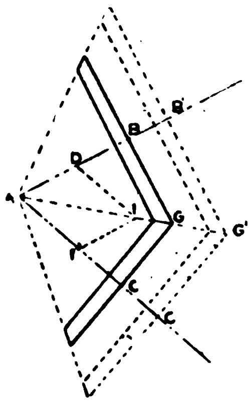
The profile Af (Fig. p. 113) of the cutter is made to correspond to the exact profile of one side of the space at I (Fig. p. 112), that is, at the middle of the tooth. In the gear-cutting machine the bevel wheel is fixed and the cutter is so placed on its arbor that its point A (Fig. p. 113) is always moving along the conical element BIC (Fig. p. 112) toward the vertex C. In order that this may obtain for each side of the space, the teeth are first all cut on one side and then the cutter is shifted on its arbor an amount equal to its thickness $AA = CC$ (Fig. p. 113) at the pitch circle; the remaining sides are then cut. The surfaces of the teeth are therefore exact where they intersect the surface of the pitch cone, and differ most from the theoretically correct forms at the tops of the end sections of the teeth.

Instead of employing the approximate method of forming the tooth profiles shown in table it will be somewhat more accurate to lay off the profiles by means of the rectangular coordinates given in tables. In case this is done it should be remembered that the diameter by which the tabular numbers are to be multiplied is not BD, the diameter of nominal pitch circle of the bevel wheel, but $2 \times IH$ (Fig. p. 112) $= 2 \times CA$ (Fig. p. 113) $=$ diameter of the developed pitch circle; from this it follows that the action of the teeth in any bevel wheel is equivalent to that of a spur wheel of the same pitch whose radius $IH = IC \tan ACB = AB \times \frac{IC}{AC}$ (Fig. p. 112). Whenever, as is generally the case, $IC > AC$ we will have $IH > AB$, and the bevel wheel will be equivalent in its action to a spur wheel having a greater number of teeth. Since spur wheels act better the greater number of teeth they possess we have here a reason for the superior action of bevel wheels over spur wheels of the same number of teeth.

It is not customary to construct bevel wheels as interchangeable wheels, partly because comparatively few of them are employed, but also because more conditions are to be fulfilled than in the case of spur wheels. In the latter the same pitch and the same

generatrix for the profiles are the only conditions; in bevel wheels, however, two other conditions are added of which the first is, the elements of the pitch cones measured from the common vertex must be of the same length; for instance, all of the pitch cones shown in the adjacent figure satisfy this condition: and the second (extra) condition is that the half sum of the angles at the vertices of a pair of pitch cones must equal the angle included between the axes. Interchangeable bevel wheels would be of little value in practice.





We can readily determine the pitch cones when the position of the axes and the velocity ratio are given, by means of the following graphical method. Let AB and AC be the axes intersecting in A. Through any point D in AB draw DF parallel to AC and make

$$\left.\begin{array}{l}\dfrac{DF}{AD} = \dfrac{\text{angular velocity of } AB}{\text{angular velocity of } AC} = \\ \dfrac{\text{rev. per min. of } B}{\text{rev. per min. of } C} = \text{velocity ratio of } B \text{ to } C;\end{array}\right\} \quad (198)$$

join AF, then will AF be the line of contact of the two pitch cones and the required frustra or bevel wheels may be cut from these cones at any convenient distance from the common vertex.

That this will give the required velocity ratio is evident from the equation,

$$\frac{DF}{AD} = \frac{\sin DAF}{\sin AFD} = \frac{\sin DAF}{\sin FAC} = \frac{BG}{CG} = \frac{B'G'}{C'G'}, \quad (199)$$

FORMULAS RELATING TO THE STRENGTH AND DURABILITY OF THE TEETH.

The symbols in the following formulas have the same meaning as in the case of spur wheels, pp. 72 to 74, the subscript $_m$ referring to the mean pitch, radius, diameter, etc., of the bevel wheel:

$$f = \frac{160800}{v_m + 36} = \frac{4667}{1 + 0.0001212 d_m n}, \tag{200}$$

$$bp'_m = \frac{1.68 P_m}{f}, \tag{201}$$

$$\frac{N}{n} = \frac{z}{Z} = \frac{d}{D} = \frac{d_m}{D_m}, \tag{202}$$

$$P_m R_m = 63025 \frac{H}{N}, \tag{203}$$

$$P_m r_m = 63025 \frac{H}{n}, \tag{204}$$

$$p'_m = 4.1 \sqrt{\frac{P_m}{f} \frac{p'_m}{b}} = 1455 \sqrt{\frac{H}{fnd} \frac{p'_m}{b}} = 188 \sqrt[3]{\frac{H}{fnz} \frac{p'_m}{b}}, \tag{205}$$

$$\left.\begin{aligned} & b \geqq 0.0000357 P_m n; \text{ or } b \geqq 4\tfrac{1}{2} \frac{H}{d_m}, \\ & \text{or } b = \frac{126050}{A} \frac{H}{d_m} = \frac{396000}{A} \frac{H}{z p'_m}, \end{aligned}\right\} \tag{206}$$

$$d = \left(1 + \frac{b}{\sqrt{d^2_m + D^2_m}}\right) d_m$$

= diameter of nominal (largest) pitch circle in pinion. (207)

$$D = \left(1 + \frac{b}{\sqrt{d^2_m + D^2_m}}\right) D_m$$

= diameter of nominal (largest) pitch circle in wheel. (208)

The circular pitch p' on the nominal pitch circle can now easily be obtained from

$$p' = \frac{d}{d_m} p'_m = \frac{D}{D_m} p'_m = \left(1 + \frac{b}{\sqrt{d^2_m + D^2_m}}\right) p'_m; \tag{209}$$

in like manner p'_0 (pitch on smallest pitch circles of the wheels;

$$p'_0 = \left(1 - \frac{b}{\sqrt{d^2_m + D^2_m}}\right) p'_m. \tag{210}$$

The formulas for the rims, arms, hubs and shafts are either the same as, or can be readily obtained from, the corresponding formulas for spur wheels.

FIG. 1.—Assume that the pair of bevel wheels to be drawn transmits 25 horse-powers, the bevel pinion making about 300 revolutions per minute, and the bevel wheel about 120 revolutions per minute. Take $z =$ or > 25.

The number of revolutions here given need not be strictly adhered to, for we wish to introduce an odd tooth, called the hunting cog, into one of the wheels so that the same pair of teeth shall not be too frequently in contact.

Let the teeth be drawn with involute profiles.

Figs. 1, 2, 3 are to be drawn as shown in plate on as large a scale as sheet will permit. In Fig. 2 the inner end of tooth should project more from rim of wheel than shown on plate, so that inner root circle will not lie on end plane of wheel.

Figs. 4 and 5 are the development of the teeth on the outer and inner pitch circles of the bevel wheels. These figures of the teeth may then be cut out of thin flexible metal and bent round on to the heel of the flank; that is, on to the (tangent) cone which is employed in the development already described: the figures drawn on plate show too much backlash.

Fig. 6.—*Cutter for Teeth of Pinion.*—It is to be drawn as shown in plate according to *Prof. Richards'* method, p. 113, the profiles being laid off, however, by means of table of rectangular coordinates and then by a series of trials an average radius r may be found which will give an arc that will pass through the points established. The coordinates x and y of the corresponding center of curvature may next be found and laid off.

Fig. 7.—*Cutter for Teeth of Wheel.*—To be drawn in a manner similar to that already described for Fig. 6.

An interesting article by *Mr. G. B. Grant* on the limiting numbers of involute bevel gear teeth is given in Am. Machinist, February 4, 1892.

PLATE X.

WORM GEARING.

Symbols without subscript refer to worm.

Symbols with subscript refer to worm wheel.

b' = breadth of wheel.

f = permissible working stress of material in pounds per □″.

H = number of horse-powers transmitted.

n and n_1 = revolutions per minute of worm and wheel respectively.

$p''' = \text{divided axial pitch of worm} = \dfrac{\text{total axial pitch of each thread}}{\text{number of threads.}}$

$p'_1 = p''' = \text{circular pitch of worm wheel} = \dfrac{2\pi R_1}{Z_1}.$

$p'' = p''_1$ = divided normal pitch on worm and wheel respectively.

P = resistance to turning at circumference of pitch circle of wheel.

Q = force necessary to turn worm if there were no friction between it and the wheel, the lever arm being R.

Q' = actual force necessary to turn worm, the lever arm being R.

R = radius of pitch cylinder of worm.

R_1 = radius of pitch cylinder of worm wheel.

Z and Z_1 = number of threads (or teeth) in worm and wheel respectively.

γ = angle made by tangent to helix (on pitch cylinder of worm) with any plane normal to axis of worm.

γ_1 = ditto for wheel.

$\eta = \dfrac{Q}{Q_1}$ = efficiency of worm gearing.

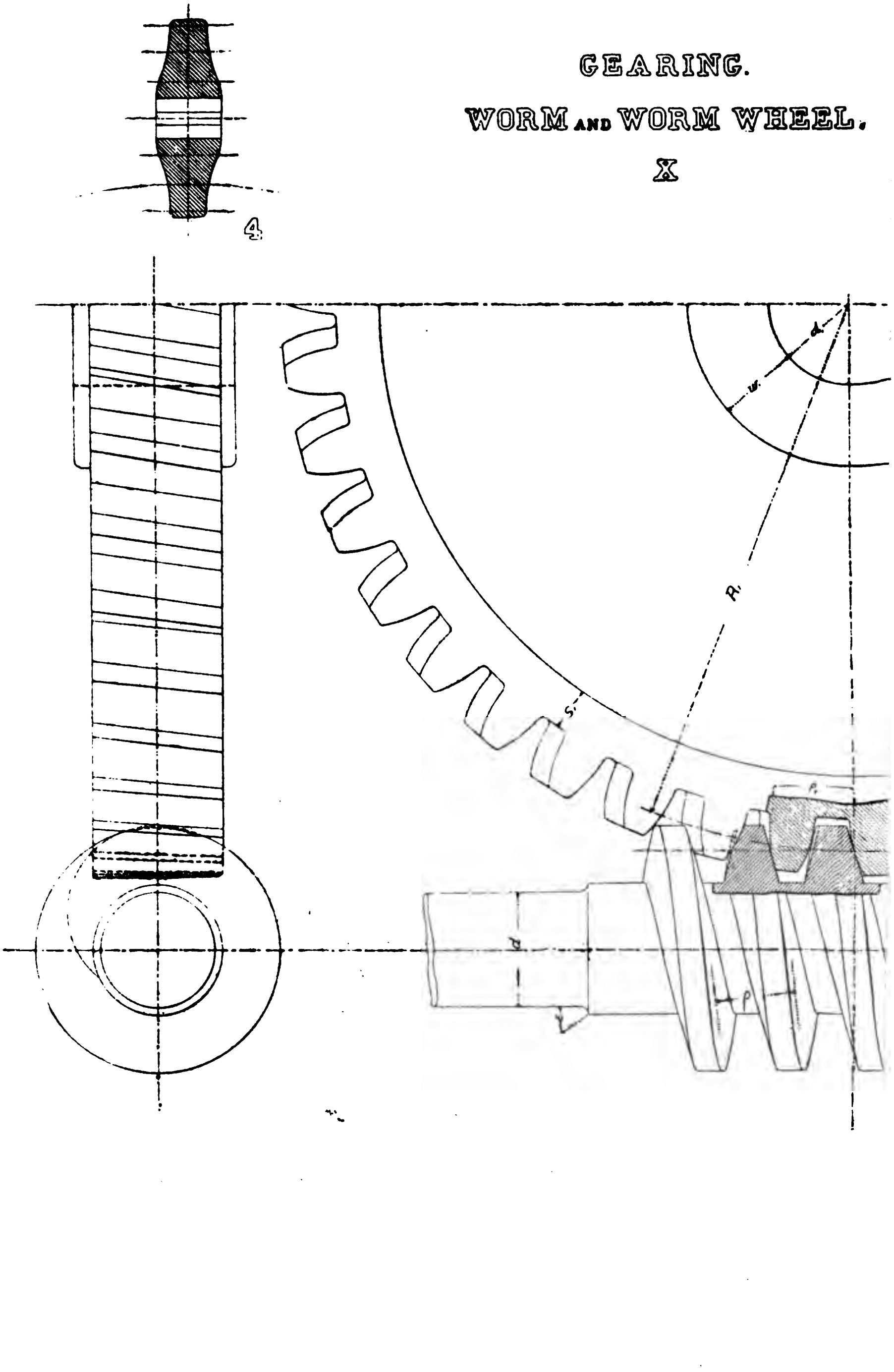
ρ and ρ_1 = radius of curvature of normal helix on pitch cylinder of worm and wheel respectively.

GEARING.

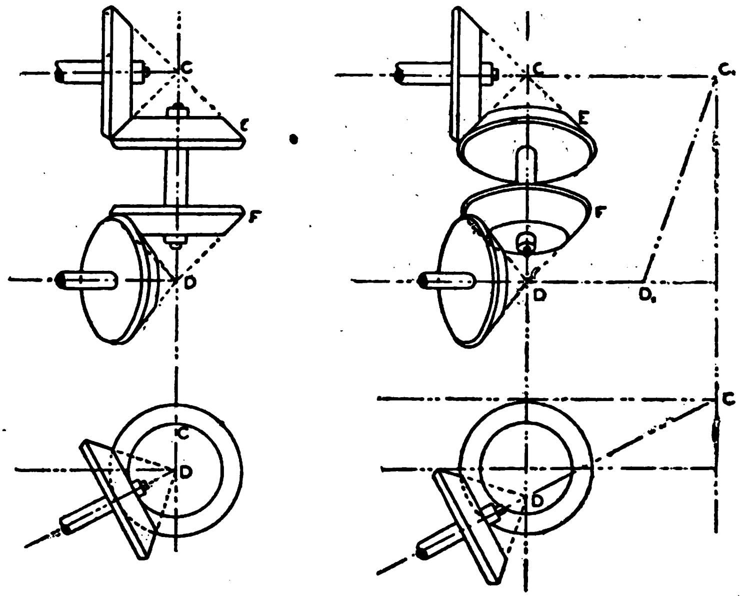
WORM AND WORM WHEEL.

X

4



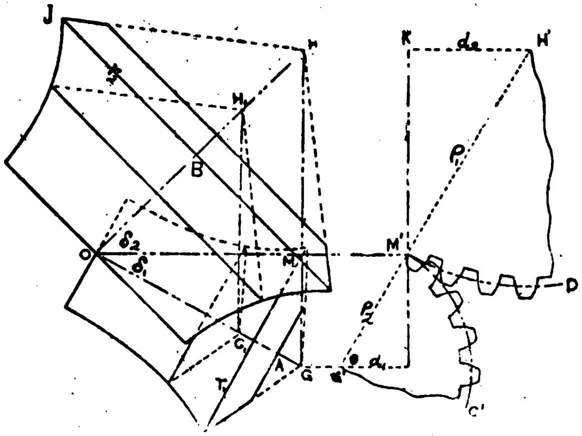
When the two axes of the wheels are neither parallel nor intersect, the axodes representing the relative motion of one wheel to another when a uniform velocity ratio is transmitted, can be shown to be hyperboloids of revolution of one nappe (see "Machinery of Transmission," *Weisbach,* Vol. III, pp. 403-410). By twisting an auxiliary hyperboloid on these primary ones, surfaces can be generated which will be suitable for the profiles of the teeth. The toothed wheels thus obtained are known as hyperboidal wheels, and are also often called skew bevel gears.



When correctly formed the teeth will be in contact along a straight line, and are then suitable for transmitting large forces; they are accompanied, however, by considerable friction and on this account and because they are difficult to make are generally avoided in practice. Shafts are often connected by bevel gears, as shown in preceding figure.

It will be well to choose the first arrangement shown, because then both pairs of wheels connect shafts which are at right angles.

For delineating the profiles an approximate method somewhat analogous to that employed for bevel gears is used; it is shown in the following figure.



The teeth are drawn on the developments of conical surfaces which are at right angles to the surfaces of the hyperboloids. In the figure the axes OH and OG are parallel to the plane of the paper; if through the points M and M_1 of the instantaneous axis OM we pass planes normal to this axis, they will intersect the axis OH and OG in the points H and G, which serve as the vertices of the auxiliary cones whose developments are $H'\ M'\ D'$ and $G'\ M'\ C'$. It is not necessary to go through the whole of the construction just given when we know the diameters d_1 and d_1 of the smallest (gorge) circle on the hyperboloids, the angles δ_1 and δ_2 made by common elements of contact (or instantaneous axis) OM with the axis and know the length l of OM; for we may make use of the formulas,

$$G'\ M' = \rho_1 = \sqrt{d_1^2 + l^2 \tan^2 \delta_1}, \qquad (211)$$

$$H'\ M' = \rho_2 = \sqrt{d_2^2 + l^2 \tan^2 \delta_2}, \qquad (212)$$

For other details and proportions see *Reuleaux'* "Konstrukteur," pp. 549-554.

Screw Gearing.

Screw gears have helicoidal surfaces for the teeth and can be formed on a lathe or screw-cutting machine. They have a velocity ratio of rotation which is independent of the radii of their pitch cylinders; this constituting one point of difference between them and the skew bevel wheels. For the general cases, as well as for the special and approximate cases of screw gearing, *Reuleaux'* "Konstrukteur," pp. 554-568, may be consulted; also *Unwin,* pp. 349-368. We will here give the formulas for the special and much used case of worm gearing only.

Worm Gearing is that special case of screw gearing in which the axes of the two wheels are at right angles and do not intersect. It is often employed when a great velocity ratio is to be transmitted with as few gears and in as small space as possible. The intersection of cylinders having the same axes as the worm and wheel will intersect the teeth in the helices. These helices have all the same axial pitch; that is, the distance measured (on each wheel, parallel to the axis, between any two successive coils, is everywhere the same. *The normal pitch* of all helices lying on the same cylinder is also the same, the normal pitch being understood to mean the distance from one coil to the next, measured, not parallel to the axis, but along the shortest line on the cylindrical surface between the two coils; that is, along another helix which cuts all the coils of the original helix at right angles.

The term *divided pitch* is made use of when a screw has more than one thread, in which case the distance between any one coil of any thread and the next coil of the same, is divided by the other threads into as many parts as the total number of threads. In that case the distance from a point on one thread to the corresponding point in the next thread is called the *divided pitch,* to distinguished it from the distance between two successive coils of the same thread, or pitch proper, which may be designated as the total pitch.

In all screw gearing the normal pitch of one wheel equals that on the other.

According to *Reuleaux,* if when the shafts are at right angles the choice of the angles γ and γ_1 is determined by the condition that the coinciding tangents of the two helices shall remain as closely together as possible both shortly before and after coincidence, the resulting gears will, with good tooth profiles, run smoothly, but they will have small bearing surfaces and will moreover require large wheels whenever the velocity ratio differs considerably from unity.

If this condition as to tangential coincidence is dropped, however, these difficulties may be avoided and the wheels still remain serviceable. But then in order that they may run exactly (*a*) each of the wheels must be constructed with slight deviations from the exact screw form, or (*b*) one wheel may be made an exact screw and the other receive the deviations, or (*c*) wear may be allowed to modify the original, approximate, screws till they become serviceable.

Procedure (*b*) is the one employed for the more careful constructions, the wheel being cut by special milling tool known as a hob, this being an exact helical steel worm notched so as to form a cutting tool. This tool is then brought into contact with the worm wheel till the teeth of the latter are envelopes of the threads or teeth of the worm; thus produced the wheel teeth differ considerably from those of perfect screw wheels. As the hob acts well only when the angle γ is small this method is not always applicable.

$$p'' = p_1'', \tag{213}$$

$$\gamma + \gamma_1 = 90^\circ, \tag{214}$$

$$\frac{Z}{Z_1} = \frac{n_1}{n} = \frac{R}{R_1}\tan\gamma, \tag{215}$$

$$\tan\gamma = \frac{Zp'''}{2\pi R} = 0.15915\,\frac{Z}{R}p''', \tag{216}$$

$$\tan \gamma_1 = 0.15915\frac{Z_1}{R_1}p_1''', \tag{217}$$

$$\frac{1}{\eta} = \frac{Q'}{Q} = \frac{1 + \varphi\dfrac{2\pi R}{Zp'''}}{1 - \varphi\dfrac{Zp'''}{2\pi R}}, \tag{218}$$

When coefficient of friction $\varphi = 0.16$ then

$$\frac{1}{\eta} = \frac{Q'}{Q} = 1 + \frac{R}{Zp'''} \tag{219}$$

$$\rho = \frac{R}{\sin^2 \gamma} \quad \rho_1 = \frac{R_1}{\sin^2 \gamma_1} = \frac{R_1}{\cos^2 \gamma} \tag{220}$$

Using same formula for calculating worm wheel that was used for spur wheels, and adapting symbols:

$$b_1p_1'' = \frac{16.8P}{f} = b_1p''. \tag{221}$$

Assuming $\qquad b_1 = 2p'''$ and $f = 10000, \qquad (222)$

we get

$$p''' = .029\sqrt{\frac{P}{\cos \gamma}} = .174\sqrt[3]{\frac{PR_1}{Z_1 \cos \gamma}} = .693\sqrt[3]{\frac{H}{n_1Z_1 \cos \gamma}}. \tag{223}$$

Proportions of rim, arm, etc., same as for spur wheels.

Assume twisting moment on worm wheel shaft $= PR_1 =$ 3000 inch pounds, $n = 800$ revolutions per minute, $n_1 = 20$ revolutions per minute.

Find the values of R and p''' which correspond to the greatest efficiency consistent with strength of worm shaft.

FIGS. 1, 2, 3.—The profiles of the teeth are designed as if for spur wheel and rack, the spur wheel having the same radius as the worm wheel; but this will give only point contact which will not ensure durability when pressures between teeth are great. The cycloidal or involute type may be employed as in spur gearing. The involute form is most desirable, because of its simple form, being a straight line for the rack. The screw or worm can easily be cut on a lathe, and in careful work not only

is this done, but as already stated, in order that the wheel teeth may be of durable shape, a steel tooth similar to the worm is also formed, parts of the threads being removed so as to form cutting edges; this steel tool or hob is then brought into contact with the worm wheel till the teeth of the latter are envelopes of the threads or teeth of the worm. When wheel is finished the steel tool is removed and the worm substituted.

When the angle γ is too great to permit the use of the hob, the wheel tooth must be formed so that its sections, in planes at right angles to its own axis, shall be envelopes to the corresponding tooth sections of the worm in these same planes. The graphical method of doing this is fully given by *Unwin* in his "Machine Design," pp. 358–368.

FIGS. 4 AND 5.—*Section of Cutter for Teeth of Worm Wheel and Worm.*—According to *Reuleaux* another method of delineating and cutting the teeth of worm and wheel may be employed.

The delineation is effected as for spur wheels, the radius of the pitch circle being ρ and ρ_1 respectively and the pitch of the teeth on this pitch circle equal to the divided normal pitch $p'' = p_1''$.

The space between the teeth thus formed will be the section of a rotary cutter, similar to those employed for spur and bevel wheels. When the teeth are being cut on either the worm or wheel, the axis of the tool remains stationary, while the piece in which the teeth are being cut both rotates about and slides along the axis. During the cutting the axis of the rotary cutter makes the angles γ and γ_1 respectively with the axis of worm or wheel.

x_1 and y_1 are the coordinates of the center of arc of approximation.

GE

PULLEY &

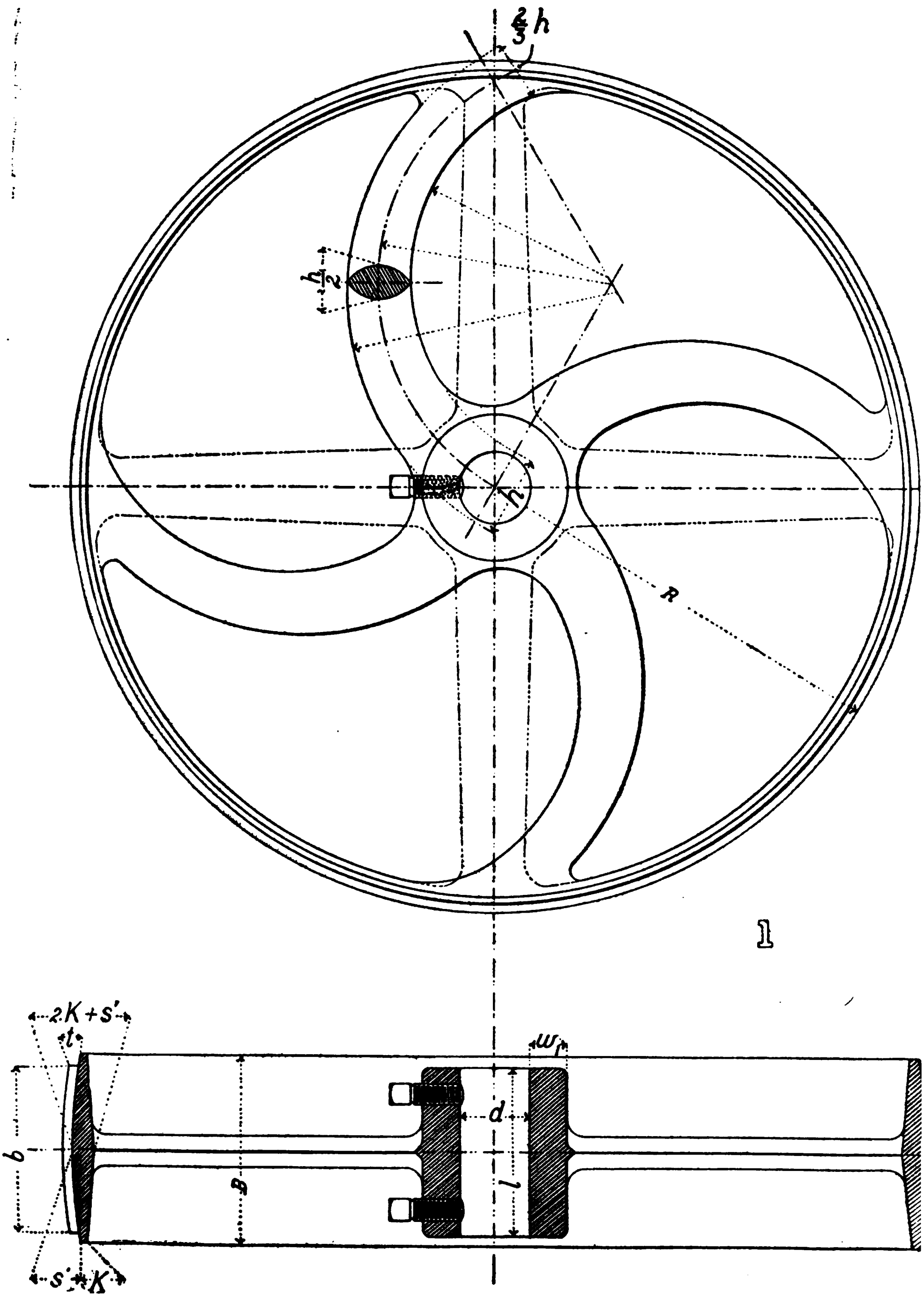


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PLATE XI.

BELT GEARING.

b = breadth of belt.
B = width of pulley rim.
d = diameter of journal shaft.
d' = diameter of rope.
D = diameter of pulley in inches.
D' = diameter of pulley in feet.
e = base of hyperbolic logarithms = 2.718—
f = allowable working stress per □″ of belt.
$F = T_1 - T_2$ = friction of belt in pounds (nearly).
g = acceleration of gravity in feet per second.
h = width of arm when continued to central plane of wheel.
H = number of horse-powers transmitted.
H_0 = number of horse-powers transmitted per square inch of belt and per 1000 feet per minute of belt speed.
K = thickness of pulley at edge.
l = length of hub of pulley.
m = abbreviation for $\frac{T_1}{P}$
n = revolutions per minute of pulley under consideration.
P = force transmitted in pounds.
Q = tension in an element of the belt in contact with the pulley.
R = radius of pulley in inches.
s = sectional area of belt.
s' = amount of swelling on crown of pulley.
t = thickness of belt.
T_1 = total tension on driving side of belt.
T_2 = total tension on slack side of belt.

T = average tension of belt $= \dfrac{T_2 + T_2}{2}$.

u = pressure exerted by an element of the belt against the surface of the pulley.

v = velocity of belt in feet per second.

V = velocity of belt in feet per minute.

V_1 = velocity of belt in 1000 feet per minute $= \dfrac{V}{1000}$.

w = thickness of metal around eye of wheel.

Z = centrifugal force of an element of the belt.

$z = 12 \dfrac{\gamma v^2}{g f}$.

α = arc of contact (expressed in circular measure), taken on smaller pulley when open belt is used and the two pulley faces are of the same material and finish.

γ = heaviness of a cubic inch of leather.

θ = number of arms.

$\rho = \dfrac{T_1}{T_2}$ = ratio of tensions in tight and slack sides of belt.

φ = coefficient of friction for sliding of belt on pulley.

$\varphi' = (1 - z)\varphi$ = a reduced value of the coefficient of friction which is equivalent to taking account of the centrifugal force of belt.

As *Prof. Reuleaux* has recently greatly simplified the discussion of this whole subject, we shall closely follow his presentation.

When a tension organ passes over a curved guide considerable friction is usually developed. Let us suppose this surface to be a pulley and that T_1 is the tension in the driving side and T_2 that in the driven side of the rope or belt. Then the friction F called forth will be,

$$F = T_1 - T_2. \tag{224}$$

This friction depends upon the arc of contact α between belt and pulley, upon the coefficient of friction φ and upon the speed v of the tension organ; it is nevertheless independent of the

radius R of the pulley. To ascertain the influence of these conditions, let Q represent the tension in an element of belt in contact with pulley (see *Weisbach-Herrmann's* "Machinery of Transmission," p. 254). The centrifugal force of an element of the belt is:

$$Z = 12\frac{\gamma s}{g}v^2 da = 12\frac{\gamma Q}{gf}v^2 da = zQda. \qquad (225)$$

The pressure u against the pully is diminished by this force Z and as the difference in the tension of two consecutive elements of the belt is just equal to the friction we have,

$$dQ = \varphi\,(u - Z).$$

Substituting we have,

$$\left.\begin{aligned} dQ &= \varphi\left(Qda - 12\frac{\gamma}{g}\frac{Q}{f}v^2 da.\right) \\ &= Q\left(1 - 12\frac{\gamma}{g}\frac{v^2}{f}\right)\varphi da \doteq Q(1 - z)\varphi da. \end{aligned}\right\} \qquad (227)$$

Transforming and integrating we get,

$$\int_{Q=T_2}^{Q=T_1}\frac{dQ}{Q} = \int_o^a \varphi(1 - z)da = \varphi(1 - z)a = \text{hyp. log.}\frac{T_1}{T_2}, \quad (228)$$

from this we get,

$$\frac{T_1}{T_2} = e^{\phi a(1 - z)} \text{ and } T_1 = T_2 e^{\phi a(1 - z)}. \qquad (229)$$

The permissible working stress f entering into

$$z = 12\frac{\gamma}{g}\frac{v^2}{f} = \frac{1}{300}\frac{\gamma}{g}\frac{V^2}{f}, \qquad (230)$$

is really also a function of a but in the above work was treated as a constant, possessing an average value corresponding to the average value of a. Taking for leather, hempen and cotton rope, $\frac{\gamma}{g} = \frac{1}{1000}$ and the stress f in the wire rope as 9 times that of hempen and cotton rope, we get the following table:

VALUE OF COEFFICIENTS $1 - z$ FOR THE CENTRIFUGAL FORCE.

Belts and Ropes. Permissible working stress, f.	Circumferential velocity in feet per minute $= V$.							Wire Rope. Permissible working stress f_1.
	1000	2000	3000	4000	5000	6000	7000	
200	.983	.933	.850	.733	.583	.400	.183	1800
300	.989	.956	.900	.822	.722	.600	.456	2700
400	.992	.967	.925	.863	.792	.700	.592	3600
500	.993	.973	.940	.897	.833	.760	.700	4500
600	.994	.978	.950	.911	.861	.800	.728	5400
800	.996	.983	.963	.933	.896	.850	.796	7200
1000	.997	.987	.970	.947	.917	.880	.837	9000
1200	.997	.989	.975	.956	.931	.900	.864	10800
1400	.998	.990	.979	.962	.940	.914	.883	12600

The table shows that high tensions are needed at high speeds to meet the disturbing effect of the centrifugal force. To take account of the latter readily we can place $\varphi a(1 - z) = \varphi' a$ regarding φ' as a sort of coefficient of friction equai to the product of $1 - z$ and the actual coefficient of friction φ.

If we neglect the stiffness of tension organ and resistance of the air we can put

$$F = T_1 - T_2 = T_2(e^{\varphi' a} - 1) = P, \qquad (231)$$

where P will be the minimum value of the force transmitted. Transforming this equation, we have

$$\frac{T_1}{T_2} = \rho = e^{\varphi' a}, \qquad (232)$$

also

$$\frac{T_1}{P} = m = \frac{e^{\varphi' a}}{e^{\varphi' a} - 1} = \frac{\rho}{\rho - 1}. \qquad (233)$$

Values of ρ and m.

φ'a	ρ	m	φ'	ρ	m	φ'a	ρ	m
0.1	1.11	10.41	1.1	3.00	1.50	2.2	9.03	1.13
0.2	1.22	5.52	1.2	3.32	1.43	2.4	11.02	1.10
0.3	1.35	3.86	1.3	3.67	1.37	2.6	13.46	1.08
0.4	1.49	3.03	1.4	4.06	1.33	2.8	16.44	1.07
0.5	1.65	2.54	1.5	4.48	1.29	3.0	20.09	1.05
0.6	1.82	2.22	1.6	4.95	1.25	3.2	24.53	1.04
0.7	2.01	1.99	1.7	5.47	1.22	3.4	29.96	1.03
0.8	2.23	1.86	1.8	6.05	1.20	3.6	36.60	1.03
0.9	2.46	1.69	1.9	6.69	1.18	3.8	44.70	1.02
1.0	2.72	1.58	2.0	7.39	1.16	4.0	54.60	1.02

CROSS-SECTION AND SPECIFIC DUTY OF A BELT.

The cross-section of a belt having the breadth b and thickness t is determined by the consideration that it must be capable of resisting the maximum tension T_1 in the driving or tight side of the belt. Then,

$$T_1 = btf. \tag{234}$$

The relation between horse power H transmitted and belt section is given by,

$$H = \frac{Pv}{550} = \frac{PV}{33000} = \frac{btfV}{33000m} = \frac{btfV_1}{33m}. \tag{235}$$

The horse-power H_0 transmitted per square inch and unit of speed (1000 feet per minute) is expressed by,

$$H_0 = \frac{1}{33}\frac{f}{m}. \tag{236}$$

This is called by *Reuleaux* the specific duty of a belt; it possesses the peculiarity of depending only on the material of the belt and pulley, provided the arc of contact α remains the same; this dependence evidently holds for the working stress f and as m is a function of the arc α and coefficient of friction φ, it is also true of m that it depends only on material of belt and pulley

This conception of a specific duty of a belt is most valuable.

because it is a direct measure of the excellence of a belt; it is to be hoped that all belts and ropes will soon be rated and classed according to this standard. The term is justified by its analogy to the term specific gravity.

The cross-section of the belt is now readily expressed in terms of horse-power, speed and specific duty, thus,

$$bt = \frac{H}{H_0 V_1}. \tag{237}$$

It is usually assumed in American discussions of belting problems that belts of single thickness are always $\frac{7}{32}$ inch thick; but in reality the thickness may vary as much as 25 % on each side of this value. To determine H_0 *Reuleaux* assumes for the different belting materials the following values for f, φ, ρ, and m, the values for φ being taken for *new* belts on smooth cast iron pulleys, while those for ρ and m are for speeds varying from 0 to 3000 feet per minute and an average arc of contact α equal to 0.95π.

Leather, $f = 440$ to 640; $\varphi = 0.16$ to 0.25; $\rho = 1.6$ to 2.1; $m = 2.5$ to 1.9 (238)

Cotton, $f = 280$ to 440; $\varphi = 0.16$ to 0.25; $\rho = 1.6$ to 2.1; $m = 2.5$ to 1.9. (239)

Rubber, $f = 360$ to 500; $\varphi = 0.20$ to 0.25; $\rho = 1.8$ to 2.1; $m = 2.2$ to 1.9. (240)

These values give for the specific duty H_0

$$\text{For leather } H_0 = 5.3 \text{ to } 9.8. \tag{241}$$

$$\text{For cotton } H_0 = 3.6 \text{ to } 6.9. \tag{242}$$

$$\text{For rubber } H_0 = 5.0 \text{ to } 8.2. \tag{243}$$

The strength of a belt is of course that of its weakest place, the laced or riveted joint. According to *Mr. Henry Towne's* experiments a riveted joint has an ultimate strength of 1750

pounds and a laced joint of 960 pounds, per square inch. Assuming a factor of safety of $\frac{1}{3}$ the permissible working stress per square inch f will be about 300 pounds. For riveted joints, we may take $f = 600$ pounds.

The coefficient of friction φ for belts ordinarily tight and having the average slip usual in practice was found by the experiments at the Massachusetts Institute of Technology to be about 0.27. Assuming as above an average speed of 1500 feet per minute and an average arc of contact $= 0.95\pi$ on smaller pulley, we get $1 - z = 0.975$; hence $\varphi' = 0.975 \times 0.27 = 0.263$ and $\varphi'\alpha = 0.786$; from this we find by the table $\rho = 2.2$ and $m = 1.88$ and finally,

$$H_0 = 4.84 \text{ for a belt speed of 1000 feet per minute.} \tag{244}$$

If we also assume the average thickness $t = \frac{7}{32}$ we get for width of belt,

$$\frac{H}{b} = 1.06 V_1;\ b = \frac{H}{1.06 V_1}. \tag{245}$$

For an arc of contact of 180° we have $H_0 = 5$ exactly for belt with laced joint. For $a° = 180°$ and riveted joint, $H_0 = 10$ exactly, which is equivalent to one horse-power for every 100 feet per minute of speed and every square inch of belt section.

For lists of executed belt transmissions and a convenient table for taking arc of contact into account see paper by Mr. Nagle in Trans. Am. S. M. E., Vol. II, p. 96; also *Reauleaux' Konstrukteur,* p. 773.

It is evident that as H_0 varies with m and the latter with the speed, we must for high speed either determine a new value for H_0, or what is better still, go back to the general formulas (229), (233) and (235). This return to the original, general, equations will also be necessary, when, as in the case of wooden or paper pulleys the coefficient of friction is very different, when the angle of contact a is > than 180° and when the belts are of unusual thickness.

From equation (236) we have for all kinds of tension organs:

$$H_0 = \frac{1}{33}\frac{f}{m}. \qquad (246)$$

For ropes, m depends not only upon the coefficient of friction φ but also upon the form of the pulley grooves in which they lie. If this groove is semi-cylindrical the friction developed is but slightly greater than if the rope rested on an ordinary cylindrical pulley rim; but if the pulley is wedge or **V** shaped, the contact surface is smaller, but the adhesion greater. The influence of groove shape can therefore be taken into account by means of the coefficient of friction. According to recent careful experiments by *Leloutre* new hempen ropes resting on cylindrical pulleys with flat rims have $\varphi = 0.075$, when the pulleys have semi-cylindrical rope grooves $\varphi = 0.088$ and when they have **V** groove possessing an angle of 60°, the value of $\varphi = 0.15$, hence for ropes wrapped half way round a pulley, we have the H_0 transmitted per square inch and per 1000 feet per minute of rope speed:

Half-round grooves, $\varphi' = 0.088$; $\varphi' a = 0.300$; $m = 3.86$;
$f = 360$; $H_0 = 2.83$. (247)

V grooves, $\varphi' = 0.150$; $\varphi' a = 0.470$; $m = 2.67$;
$f = 360$; $H_0 = 4.09$. (248)

From $H = \frac{PV_1}{33}$ we have $$d' = \sqrt{\frac{1}{.7854H_0}\cdot\frac{H}{V_1}} = \frac{2}{3}\sqrt{\frac{H}{V_1}}, \qquad (249)$$

$$d' = 0.56\sqrt{\frac{H}{V_1}} \qquad (250)$$

For **V** grooves it often happens that only $H_0 = 2.$ is transmitted, then

$$d' = \frac{4}{5}\sqrt{\frac{H}{V_1}}. \qquad (251)$$

When a large amount of power is to be transmitted several ropes placed side by side are employed. In power transmissions 2-inch ropes are much used, but they are sometimes as low as 1.2 inches and as high as 2.8 inches.

For the loss of efficiency in belt transmissions, due to journal friction, stiffness of ropes and slip of belt, see *Reuleaux'* "Konstrukteur," p. 783, and *Weisbach-Hermann's* "Machinery of Transmission," pp. 244-254.

The constants for rubber belting have already been given above. An advantage of rubber is that it can be used in wet places. It should be vulcanized, otherwise slipping of the belt will soon heat and ruin it. Rubber belts have no joints, and are made of the following thickness:

$$\text{Number of ply} = \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$$
$$\text{thickness} = \tfrac{3}{16}'' \quad \tfrac{5}{24}'' \quad \tfrac{5}{16}'' \quad \tfrac{5}{12}'' \quad \tfrac{7}{16}''.$$

Arrangement of Pulleys when Shafts are not Parallel.

The following rule will be found universally applicable:

Place the adjacent pulleys so that the point at which the belt is delivered from each pulley will lie on the plane of the pulley on to which the belt is running. See *Reuleaux,* Ger. Ed., pp. 346-357; French Ed., pp. 557-569; also *Unwin,* pp. 391-394.

Pulleys having twist belts should be at sufficient distance to give an easy twist.

Whenever it is convenient the slack side of the belt should be placed on upper side, so as to increase the hug, or arc of contact.

Wooden drums and leather-covered pulleys (the latter are the better) give better results than cast-iron pulleys, *i. e.,* there is less friction on shaft for transmission of power.

Belts should have an occasional greasing with castor oil to keep them soft and pliable.

Fig. 1—*Ordinary Pulley with Straight and with Curved Arms.*

The curved arms are not so liable to crack in cooling as the straight ones. The convexity of pulley face is to prevent the belt from slipping off. It also makes it easier (with a given tension) to put on the belt.

$$s' = \frac{b}{80} \qquad (252)$$

$$B = \tfrac{5}{4}b \text{ or } B = \tfrac{9}{8}(b + 0.4). \qquad (253)$$

$$K = .01B + 0.08, \qquad (254)$$

$$\theta = \tfrac{1}{2}\left(5 + \frac{R}{b}\right) = \text{number of arms}, \qquad (255)$$

$$h = \frac{l}{4} + 0.1\frac{R}{\theta} + \tfrac{3}{8}, \qquad (256)$$

$$w = \frac{d}{6} + .02R + .4, \qquad (257)$$

$$l = \text{or} > 2.5w, \qquad l = \text{or} < B. \qquad (258)$$

$$\gamma^\circ = 90^\circ - \frac{240^\circ}{\theta}. \qquad (259)$$

Lay off a line making the angle γ° with radial line of the spoke, at the center of that portion of the radial line representing the length of arm erect perpendicular; where it intersects the γ line will be the point from which the central arc of the arm may be described; then find two other centers from which arcs can be described which will pass through points at distances $=\frac{h}{3}$ and $\frac{h}{2}$ from central arc (at extremities of arm).

Assume $H = 4.5$. $n = 125$ revelutions per minute. Speed of belt = 1000 feet per minute.

Fig. 2.—*Stepped Cone Pulleys Driven by Crossed Belts.*

$$\delta = 0.12B + .08, \qquad (260)$$

$$n' = 0.16B + .08, \qquad (261)$$

$$\delta = 0.2\sqrt{b} + \tfrac{1}{8} = \text{diameter of bolt}. \qquad (262)$$

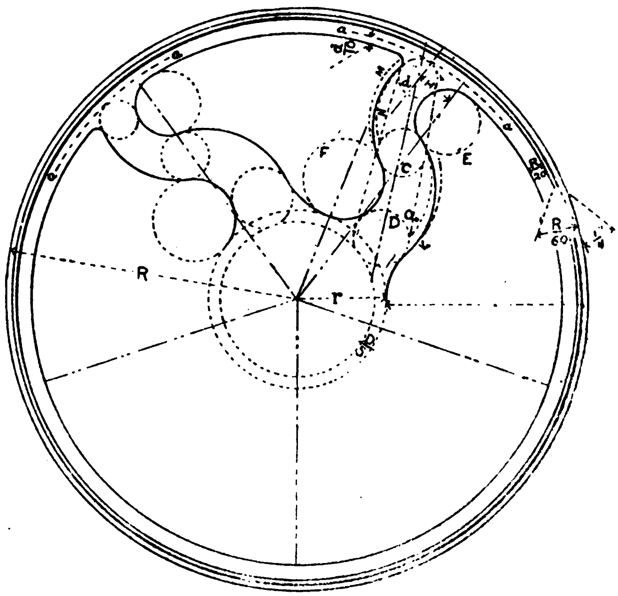
Assume $b = 4''$. $d = 3''$. Take diameter of steps on one cone as in figure, and those on its fellow as if the pair of cones were to be employed with crossed belt.

For grooved pulleys see *Unwin*, p. 341.

The symbols used in the following figure have not the same meaning as the same symbols used in Figs. 1 and 2.

FIG. 3.—*Pulley with S-Shaped Arm used in Baxter Engine.*

$$\left.\begin{array}{l} r = .3R. \quad l = R - r = .7R. \\[2ex] D = .55\sqrt[3]{P}. \quad d = \frac{2}{3}D. \quad H = \frac{R}{8}. \end{array}\right\} \qquad (263)$$



First divide the pulley by means of radii into five equal parts. Then, to construct each arm, draw the circle d so that its center will be at the distance H from the radial line, and its circumference tangent to an auxiliary circle *aa*, drawn half-way between

the inner and outer circles of the rim. Next draw the circle D so that it will touch the radial line and an auxiliary circle with radius $= r = \frac{D}{5}$. Now join the centers and draw tangents to the two circles d and D. From a point on the line of centers and half-way between the circumferences of d and D describe a circle G just touching the tangents. Draw circle E so that it will touch G, d and inner circle of rim; also draw F so that it will touch G, D and r; with a radius equal to $\frac{M + N}{2}$ describe a circle S that will touch d and G; also with a radius equal to $\frac{K + O}{2}$ describe a circle that will touch D and G.

Now draw radial line through center of circle F, then find a point on this radius equidistant from r and S and with this point as a center draw a circle tangent to both r and S.

Thickness of arm $= \frac{1}{3}D$. Breadth of face $=$ or $< 1\frac{1}{2}l$.

Arms of the above proportions were applied to pulleys in which $R =$ or $< 15''$. Assume $R = 12''$.

BELT GEARING. CONE PULLE

XII

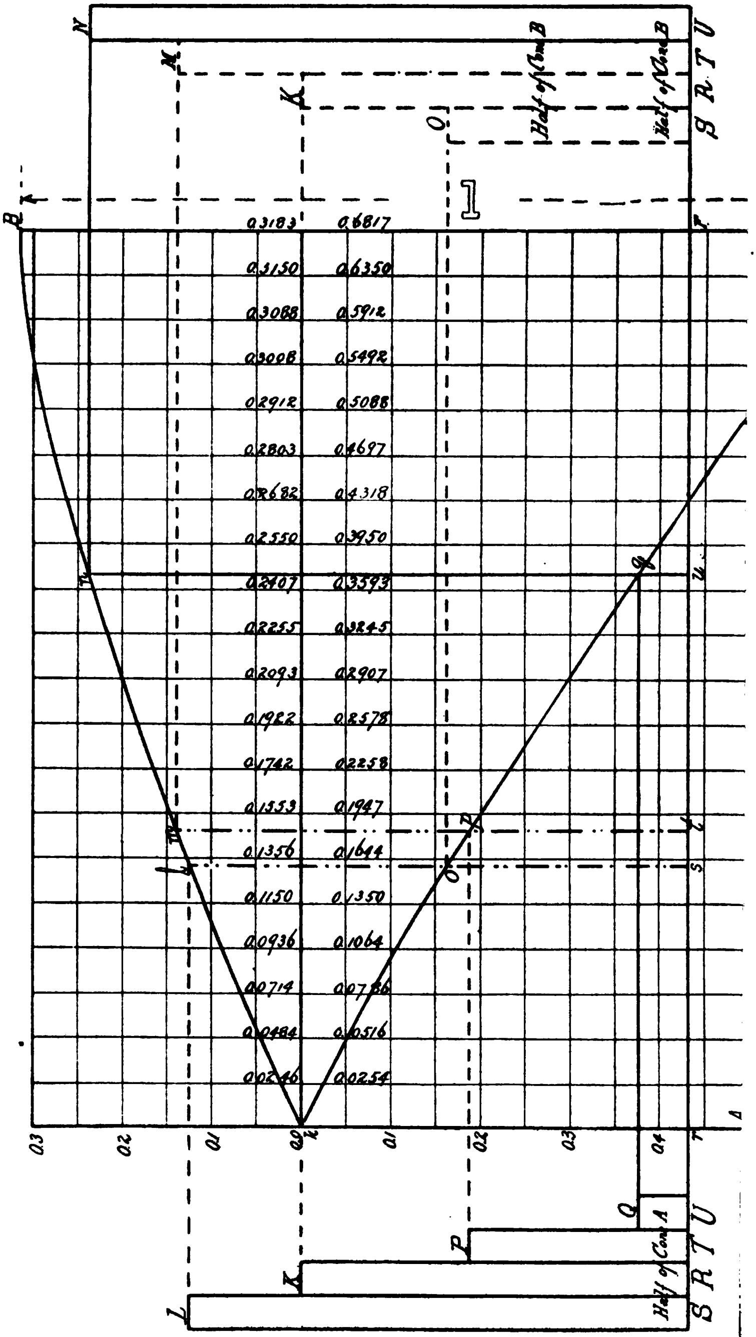


Diagram for determining the Radii of Speed Cones

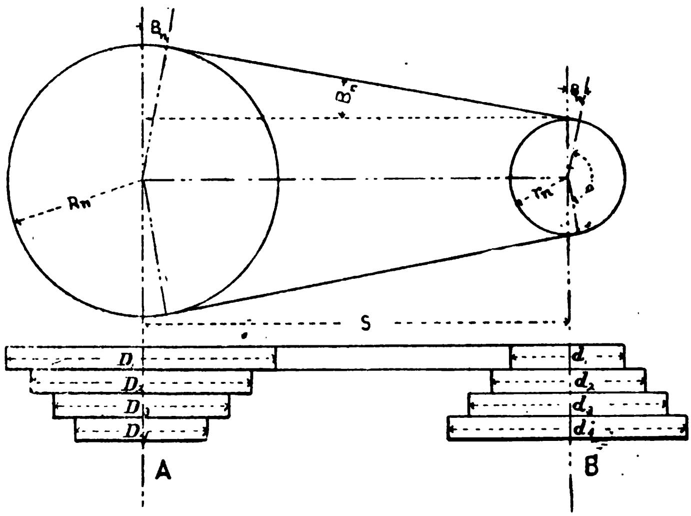
PLATE XII.

BELT GEARING.—CONE PULLEYS.

DIAGRAM AND TABLES FOR DETERMINING THE DIAMETERS OF SPEED CONES WHEN CONNECTED BY AN OPEN BELT OF CONSTANT LENGTH.

The difficulty in a short and accurate solution of this problem is the complex and transcendental nature of the equation which expresses the relation between the diameters of any belted pair of steps.

The equation itself is easily obtained from the following figure



$$\text{Thus } l = R_n\left(\frac{\pi}{2} + B_n\right) + r_n\left(\frac{\pi}{2} - B_n\right) + S \cos B_n, \quad (264)$$

$$R_n - r_n = S \sin B_n. \quad (265)$$

Here l represents the half length of the belt, R_n and r_n the radii respectively of the larger and smaller of any belted pair of steps and B_n the inclination of the straight part of the belt to line of centers.

Combining the preceding equations, we get,

$$\frac{S}{l} = \frac{\pi}{4}\,\frac{D_n + d_n}{S}\,B_n \sin B_n + \cos B_n. \qquad (266)$$

In order that the belt may have equal tension on every pair of steps, it must be of constant length, consequently

$$\left.\begin{aligned} l = R_1\left(\frac{\pi}{2} + B_1\right) + r_1\left(\frac{\pi}{2} - B_1\right) + S \cos B_1 = \\ R_2\left(\frac{\pi}{2} + B_2\right) + r_2\left(\frac{\pi}{2} - B_2\right) + S \cos B_2, \end{aligned}\right\} \qquad (267)$$

$$\left.\begin{aligned} \text{or } (R_1 + r_1)\frac{\pi}{2} + (R_1 - r_1)B_1 + S \cos B_1 = \\ (R_2 + r_2)\frac{\pi}{2} + (R_2 - r_2)B_2 + S \cos B_2, \end{aligned}\right\} \qquad (268)$$

a more general expression is,

$$\left.\begin{aligned} (R_n + r_n)\frac{\pi}{2} + (R_n - r_n)B_n + S \cos B_n = \\ (R_{n+1} + r_{n+1})\frac{\pi}{2} + (R_{n+1} - r_{n+1})B_{n+1} + S \cos B_{n+1}. \end{aligned}\right\} \qquad (269)$$

The data for determining the unknown diameters in the two cases occurring in practice are as follows:

CASE I.—*Given*, all the diameters D_1, D_2, D_3, etc., on one cone A, one diameter d_n on cone B and the distance S between centers.

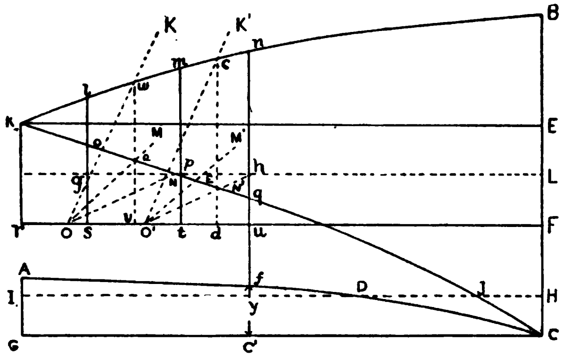
CASE II.—*Given*, the diameters D_n and d_n of one belted pair of steps, the ratios $\frac{D_1}{d_1}$, $\frac{D_2}{d_2}$, $\frac{D_3}{d_3}$, etc., of the diameters of the other pairs, and the distance S between the axes.

Each of these two problems can be solved by two different methods, namely, the graphical and the tabular.

Graphical Method.

Of all the graphical methods that have been devised for solving the speed cone problem, that of *Prof. Culmann* is the only one which is exact; it is, moreover, the only one which possesses the time-saving property of needing to be constructed but once in order to be applicable to every speed cone problem which may arise in practice. The only objection to be made to *Culman's* diagram is, that though easy of application and not difficult to understand, it nevertheless requires considerable time and care for its accurate construction. The writer has sought to remove this objection to the diagram by calculating the ordinates of the curve forming its essential feature.

The ordinates are given in Plate XII. (For method of computing ordinates, see article in *Journal Franklin Institute*, May 1880.)



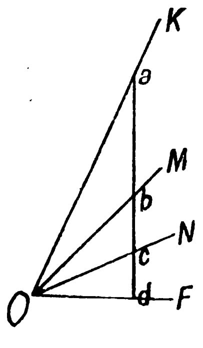
To show how the curve may be employed in solving speed cone problems we will make use of the above figure.

In figure each one of the vertical chords, as *ol*, represents the difference between the radii of some belted pair of steps, the greatest possible vertical chord, *BC*, being equal to *S*, or distance between the axes. Both the given and required radii are estimated from some horizontal line, as *rF*, which we call the datum line. The different positions of the datum line correspond to different lengths of belt.

Suppose we had given all the radii of cone *A*, one radius of cone *B*, and the distance *S* between the axes.

If we now take line *BC* as the linear representative of *S*, and to the same scale $\frac{QU}{NU}$ *PT*, *KR* and *LS*, (see Plate XII) as the linear equivalent of the given radii, we can find the required partners of *PT*, *KR* and *LS* by the following process. We first find that vertical chord *nq* of the curve *BkC*, which is equal to the difference $NU - QU$ of the radii of the given pair; from the lower end of the chord thus found we lay off *qu* equal to the smaller radius *QU* of the given pair, and through *u* draw the horizontal or datum line *ruF*. To find the companion of *PT* we have simply to find the point *p* on the curve *BkC* whose distance *pt* from the datum line *u* is equal to *PT;* then, drawing a vertical line through *p* we will have in *mt* the required radius to the scale $\frac{BC}{S}$. For the companion step to *KR* we obtain its equal, *kr*, the difference between the members of the belted pair being in this case equal to zero. For the partner of *LS* we obtain *os*.

If we had one pair of radii given, and the ratios $\frac{2}{1}$ $\frac{4}{1}$ of the remaining pairs, we would, as in the previous example, first find that vertical chord of the curve *BkC* which is equal to the difference of the given radii, and, as before, lay off the smaller radius to obtain the datum line from which radii are estimated. When the datum line has been located we can easily find the radii having the required ratios, either by trial or as follows: Draw any convenient angle, *KOF*, then bisect any line as *ad*, which is drawn perpendicular to *OF* at *b*, also *bd* at *c*; then will $\frac{ad}{bd} = \frac{2}{1}$ and $\frac{ad}{cd} = \frac{4}{1}$, and any line parallel to *ad* will be divided in the



same proportion by the rays OF, ON, OM and OK. Now if figure is drawn on some such transparent material as tracing cloth, the line OF may be placed upon and moved along the datum line rF, see p. 139, till the respective intersections of the rays OK and OM, with the upper and lower branches of the curve BkC are in same vertitcal line, as at wa, p. 139, then will wv and av be the radii having the required ratio $\frac{wv}{av} = \frac{2}{1}$. Moving line OF still farther along the datum line rF (see p. 139) we can obtain in like manner the radii cd and ed with the required ratio $\frac{cd}{ed} = \frac{4}{1}$. As the sum of the radii of the steps cannot be greater than the distance between the axes, there is evidently a limit to the size of the steps for every given difference of diameters; these limits taken together form another curve, CDA on p. 139. The vertical lines qf and nf included between this curve and two branches of BkC are the maximum radii for the given difference nq, for $qf + nf = BC = S$.

It follows from this, that only from that portion of the datum line DH which falls above the limiting curve CDA can radii be estimated. But even the portion DH can not be wholly thus used, for there is still another very evident condition, namely that the radii can never be less than zero, consequently it is only from the portion DJ of the datum line IH that radii can be estimated.

We have given in Plate XII the ordinates of the upper and lower branches of the curve BkC shown in Fig. on p. 139. The ordinates were chosen so that the chords or differences of radii should form an arithmetical series whose first term was 0 and difference 0.05. These ordinates were made equidistant on Plate XII as a matter of convenience only, for the diagram gives correct results even when the increments of the abscissas are not constant, the reason of this being that, since both data and results are vertical quantities, any amount of horizontal distortion is

permissible which allows the chords to remain vertical. The most convenient assumption is to take the abscissas as numerically equal to the vertical *chords* existing between branches of *BkC*. In laying out this curve on section paper it is not essential that abscissas and chords should then be to the same scale; but for easy finding of given chord (*i. e.*, given difference of steps) the values 0, .05, .10, .15, .20, etc., of the abscissas should begin on the main lines of the section paper. This convenient arrangement is not shown in Plate XII.

Construct the curve *BkC* of Plate XII upon finely lined section paper, and to such a scale that the greatest chord $BC =$ 40 inches; the results obtainable from this diagram will be sufficiently accurate for all the cases which arise in practice.

THE TABULAR METHOD.

The first of the three following tables is applicable to all cone pulley problems.

We will now show by a series of examples how the table is to be employed.

EXAMPLE *I.*—*Given*,

the four diametrs: 4″ 8″ 14″ 20″ on cone *A*,
one diameter — — 14″ — on cone *B* and the distance between the axes equal to 40″.

Required, the remaining diameters on cone *B*.

The given pair consisting of equal steps, we have $\frac{D - d}{S} = \frac{14 - 14}{40} = 0$, and must, therefore, look along the middle horizontal row for the diameter $\frac{14}{40} = .3500$ in order that we may find the column corresponding to the length of belt containing these particular cones,

D_n = diameter of the larger of a (belted) pair of steps, in inches.
d_n = diameter of the smaller of a (belted) pair of steps, in inches.
S = distance between the centers of two cone pulleys, in inches.

L = length of belt in inches.

The upper and lower halves of table give respectively valves of $\frac{d_n}{S}$ and $\frac{D_n}{S}$.

Difference of Diameters of Steps ÷ S $=\frac{D_n-d_n}{S}$	Values of $\frac{L}{S}$ length of Open Belt when Axes are a Unit's distance apart.											Differences.		Contact of Belt on Smaller Step
	2.31416	2.62832	2.94248	3.25664	3.57080	3.88496	4.19911	4.51327	4.82743	5.14159	5.45575			
	1	2	3	4	5	6	7	8	9	10	11	1st.	2d.	
1.5000										.0606	.1606	758		83°
1.4000									.0364	.1364	.2364	736	22	91°
1.3000								0.100	.1100	.2100	.3100	715	21	99°
1.2000								0.815	.1815	.2815	.3815	695	20	106°
1.1000							.0510	.1510	.2510	.3510	. . .	676	19	113°
1.0000						.0186	.1186	.2186	.3186	.4186	. . .	657	19	120°
0.9000						.0843	.1843	.2843	.3843	.4843	. . .	640	17	127°
0.8000					.0483	.1483	.2483	.3483	.4483	.5483	. . .	622	18	133°
0.7000				.0105	.1105	.2105	.3105	.4105	.5105	.6105	. . .	606	16	139°
0.6000				.0711	.1711	.2711	.3711	.4711	.5711	.6711	. . .	589	17	145°
0.5000			.0300	.1300	.2300	.3300	.4300	.5300	.6300	.7300	. . .	572	17	151°
0.4000			.0872	.1872	.2872	.3872	4872	.5872	.6872	.7872	. . .	556	16	157°
0.3000		.0428	.1428	.2428	.3428	.4428	.5428	.6428	.7428	.8428	. . .	540	16	163°
0.2000		.0968	.1968	.2968	.3968	.4968	.5968	.6968	.7968	.8968	. . .	524	16	169°
0.1000	.0492	.1492	.2492	.3492	.4492	.5492	.6492	.7492	.8492	.9492	. . .	508	16	174°
0.0000	.1000	.2000	.3000	.4000	.5000	.6000	.7000	.8000	.9000	. . .	. . .		16	180°
0.1000	.1492	.2492	.3492	.4492	.5492	.6492	.7492	.8492	.9492	1 0492	. . .	492	16	174°
0.2000		.2968	.3968	.4968	.5968	.6968	.7968	.8968	.9968	1.0968	. . .	476	16	169°
0.3000		.3428	.4428	.5428	.6428	.7428	.8428	.9428	1.0428	1.1428	. . .	460	16	163°
0.4000			4872	.5872	.6872	.7872	.8872	.9872	1.0872	1.1872	. . .	444	16	157°
0.5000			.5300	.6300	.7300	.8300	.9300	1.0300	1.1300	1.2300	. . .	428	17	151°
0.6000				.6711	.7711	.8711	.9711	1.0711	1.1711	1.2711	. . .	411	17	145°
0.7000				.7105	.8105	.9105	1.0105	1.1105	1.2105	1.3105	. . .	394	16	139°
0.8000					.8483	.9483	1.0483	1.1483	1.2483	1.3483	. . .	378	18	133°
0.9000						.9843	1.0843	1.1843	1.2843	1.3843	. . .	360	17	127°
1.0000						1.0186	1.1186	1.2186	1.3186	1.4186	. . .	343	19	120°
1.1000							1.1510	1.2510	1.3510	1.4510	. . .	324	19	113°
1.2000								1.2815	1.3815	1.4815	1.5815	305	20	106°
1.3000								1.3100	1.4100	1.5100	1.6100	285	21	99°
1.4000									1.4364	1.5364	1.6364	264	22	91°
1.5000										1.5606	1.6606	242		83°

The quantity .3500 evidently lies half-way between columns 3 and 4, and corresponds to a length of belt $\frac{L}{S} = \frac{2.944 + 3.258}{2} =$ 3.101. As the differences between columns 3 and 4 are everywhere .1000 we can get, if desired, another column corresponding to the length of belt 3.101, by simply adding .0500 to all the values in column 3, thus getting the series (and their differences from the table).

$\frac{D_n - d_n}{S} =$	.5	.4	.3	.2	.1	.0	.1	.2	.3	.4	.5
	.0800	.1372	.1928	.2468	.2992	.3500	.3992	.4468	.4928	.5372	.5800
Differences,	.0572	.0556	.0540	.0524	.0508	.0492	.0476	.0460	.0444	.0428	
	0016	.0016	.0016	.0016	.0016	.0016	.0016	.0016	.0016		

To get the other member of the belted pair to which the 4″ step belongs, we must look in this series for $\frac{4}{40} = 0.1000$ and will find that the next lower term is .0800. By interpolation* we shall find the corresponding value of $\frac{D_1 - d_1}{S}$ to be:

$$0.5000 - \frac{(0.1000 - 0.0800)}{.0572} \times 0.1 = 0.5000 - 0.35 \times 0.1 = 0.4650.$$

---

* This interpolation, using only the first differences, will be sufficiently accurate for all practical purposes. If for any reason greater accuracy is desired, the second differences may be introduced, using the formula,

$$xD' + \frac{x(x-1)}{2} D'' = \mathfrak{D}$$

where x is the true fraction of the tabular interval, D', D'' first and second differences and $\mathfrak{D}$ the difference between the given value of $\frac{d_n}{S}$ and the next lower value in table. In the example under consideration, we have

$$x \times 0.0572 + \frac{x^2 - x}{2} \times 0.0016 = 0.1000 - .0800 = 0.02,$$

which gives $x = 0.3528$ instead of 0.35 as above. This makes $\frac{D_1 - d_1}{S} =$ $0.5000 - 0.3528 \times 6.1000 = 0.4647$. The effect on D_1 is to give it a value of 22.588″ instead of 22.600″. For example in which x is given and $\mathfrak{D}$ sought see foot-notes on pages 148 and 149.

Adding this to $\frac{d_1}{S} = 0.1$ we get $\frac{D_1}{S} = 0.565$ and $D_1 = 0.565 \times 40 = 22.60''$.

Proceeding in the same manner for the next step we have $\frac{8}{40} = 0.2000$ and from the above series we find,

$$\frac{D_2 - d_2}{S} = .3 - \frac{(.2000 - .1928)\ .1}{.0540} = .3000 - .0133 = .2867$$

adding this to $\frac{d_2}{S} = .2000$ we have $\frac{D_2}{S} = .4867$ and $D_2 = 19.47''$ for the partner of the 8″ step.

Interpolating in like manner for the 20″ step, $\frac{24}{40} = .5$ being looked for in the series, we get,

$$\frac{D_4 - d_4}{S} = .3 + \frac{(.5000 - .4928)\ .1}{.0444} = .3162,$$

subtracting this from $\frac{D_4}{S} = .5$ we get $\frac{d_4}{S} = .1838$ and $d_4 = .1838 \times 40 = 7.35''$.

The belted pairs of the two cones are, therefore, as follows:

4	8	14	20	on cone *A*.
26.6	19.42	14	7.35	on cone *B*.

The next example differs from the one just given in having the steps of the *given* belted pair of unequal, instead of equal, diameters, and also differs in giving a shorter method of obtaining the required diameters.

EXAMPLE 2.—*Given,* the diameter 6 12 18 24 on cone *B*.
the diameter 32 — — — on cone *A*.
the distance between axes = 50″ = *S*.

Required; the unknown diameters of *A*.

Here $\frac{D_1 - d_1}{S} = \frac{32 - 6}{50} = .52$, means that we must look in the table, between the horizontal rows corresponding to $\frac{D_n - d_n}{S}$

$= .6$ and $.5$ for $\frac{d_1}{S} = \frac{6}{50} = .1200$ in order that we may find the column of the table corresponding to the length of belt on these cones.

Interpolating between the aforesaid horizontal rows we get,

$$.0300 - .0589\frac{(.52 - .50)}{.10} \doteq .0182$$

for the first term of the horizontal row corresponding to $\frac{D_n - d_n}{S} =$ .52. All the subsequent terms can now be easily obtained by successively adding .1000.

Number of column,	3	4	5	6	7	8
thus	.0182	.1182	.2182	.3182	.4182	.5182, etc.

Evidently $\frac{d_1}{S} = .1200$ lies between columns 4 and 5 at a distance from column 4 represented by $\frac{.1200 - .1182}{.2000} = \frac{.0018}{.1000} =$ .018. By adding this number .018 to the terms in column 4 we will get the series corresponding to the proper length of belt as in the previous example.

But we can also obtain the required results without finding this series, namely as follows: Look for $\frac{12}{50} - .0018 = .2382$ in column 4 and find the corresponding difference $\frac{D_2 - d_2}{S}$ of the diameters of this belted pair, by the following interpolation:

$$\frac{D_2 - d_2}{S} = .4 - \frac{(.2382 - .1872)}{.0556} \times .1 = .3083.$$

Adding $\frac{d_2}{S} = .24$ we get $\frac{D_2}{S} = .5483$ and $D_2 = .5483 \times 50 =$ 27.42. In like manner the fellow of the 18″ step may be obtained by finding the difference $\frac{D_3 - d_3}{S}$ corresponding to $\frac{18}{50}$

$- .0018 = .3582$ of column 4; thus,

$$\frac{D_3 - d_3}{S} = .1000 - \frac{.3582 - .3492}{.0508} \times .1 = .0823,$$

adding $\frac{d}{S} = .36$ we get $\frac{D}{S} = .4423$ and $D_3 = .4423 \times 50 = 22.12$.

For the 24″ step we must look for $\frac{24}{50} - .0018 = .4782$ in column 4. Its corresponding difference will be,

$$\frac{D_4 - d_4}{S} = .1 + \frac{.4782 - .4492}{.0476} \times .1 = .1609,$$

subtracting this from $\frac{D_4}{S} = .48$ we get $\frac{d_4}{S} = .3193$ and $d_4 = .3193 \times 50 = 15.96$.

The belted pairs of the speed cones will then be as follows:

6	12	18	24
32	27.42	22.12	15.96.

From the last column of the table can be obtained, if desired, the arc of contact of belt on the smaller step; for the above pairs they are as follows:

$$a = 150° \quad 162° \quad 175° \quad 171°$$

and from the top of the table we get for the length of the belt,

$$L = (3.25664 + \frac{.0018}{.1000} \times .314)\,50 = 163.11 \text{ inches}.$$

EXAMPLE 3.—The number of revolutions of speed cone A is 240, and the distance between the axes $= 40''$.

Required, that the steps be so chosen that the cone B may make 100, 240, 400 and 580 revolutions per minute.

If we assume that 30 feet per second would be a suitable maximum velocity of belt on these speed cones, and that D_1 is the diameter of the step on cone A, which transmits 580 revolutions to the step d_1 on cone B, we will get from

$$\frac{D_1}{12} < 3.14 \times \frac{240}{60} = 30,$$

$$D_1 \doteq 29'' \text{ nearly and } d_1 = \frac{240}{580} \times 29'' = 12''.$$

We can now make use of the table, first finding, as in the previous examples, the column corresponding to the length of belt connecting D_n with d_n.

Since $\dfrac{D_1 - d_1}{S} = \dfrac{29 - 12}{40} = .425$ we must look between the horizontal series corresponding to $\dfrac{D_n - d_n}{S} = .5$ and $.4$ for $\dfrac{d_1}{S} = \dfrac{12}{40} = .3000$ and will find that $\dfrac{d_1}{S}$ lies in a series obtained by adding $.3000 - [2300 + \dfrac{.5000 - .4250}{.1000} \times .0572] = .0271$ * to the values of column 5. A portion of the series thus obtained is as follows:

$\frac{D_n - d_n}{S} = .8$	.7	.6	.5	.4	.3	2	.1	.0
$\frac{d_n}{S} = .0754$	.1376	.1982	.2571	.3143	.3699	.4239	.4763	.5271

Differences = .0622 .0606 .0589 .0572 .0556 .0540 .0524 .0508
.0016 .0017 .0017 .0016 .0016 .0016 .0016

and will be sufficient if we always express the velocity ratio so that it will be greater than unity.

*In this interpolation only the first difference is used. If the greatest accuracy is desired, the second differences should be used in accordance with formula

$$xD' + \frac{(x^2 - x)}{2} D'' = \mathfrak{D}$$

given in foot-note on page 144. Substituting $x = \dfrac{0.500 - 0.425}{0.100} = \dfrac{3}{4}$,

$$D' = 0.0572 \text{ and } D'' = 0.0016, \text{ we get } \mathfrak{D} = 0.04275 \text{ and}$$
$$0.3000 - [0.2300 + 0.04275] = 0.02725$$

instead of 0.0271 as the value to be added to column 5 to get the slightly more exact series of $\dfrac{d_n}{S}$ and $\dfrac{D_n}{S}$. The effect on d_n and D_n is also insignificant, being $0.00015 \times 40 = 0.006$.

Now since the given velocity ratio

$$\frac{D_n}{d_n} = \frac{\dfrac{D_n - d_n}{S}}{\dfrac{d_n}{S}} + 1, \qquad (270)$$

we shall have,

$$\frac{D_n}{d_n} - 1 = \frac{\dfrac{D_n - d_n}{S}}{\dfrac{d_n}{S}} \qquad (271)$$

Since $\dfrac{D_2}{d_2} = \dfrac{400}{240} = \dfrac{5}{3}$, we have $\dfrac{D_2}{d_2} - 1 = \dfrac{2}{3} = \dfrac{\dfrac{D_2 - d_2}{S}}{\dfrac{d_2}{S}}$.

Looking in the above series for the position of the quotient $\dfrac{\dfrac{D_2 - d_2}{S}}{\dfrac{d_2}{S}} = \dfrac{2}{3}$ we find that it lies between $\dfrac{.3}{.3699}$ and $\dfrac{.2}{.4239}$.

Its exact position may evidently be found from the equation.

$$\frac{.3 - x \times .1}{.3699 + x \times .0540} = \frac{2}{4}.\ *$$

.*Exact interpolation would make this an equation of the second degree. For according to the interpolation formula, the second term of the denominator of this equation becomes

$$.0540x + 0.0008\,\frac{x^2 - x}{2} = \mathfrak{D}$$

and the equation ittself quadratic,

$$\frac{0.3 - x \times 0.1}{0.3699 + 0.540x^2 + 0.0008\,\frac{x^2 - x}{2}} = \frac{2}{3}.$$

Solving we get $x_2 = 0.3931$ instead of 0.3927 and then $\mathfrak{D} = 0.0209$, thus making $d_2 = 15.632''$ instead of $15.644''$ as above. If we had used the exact series of $\dfrac{d_n}{S}$ as well as the exact value of x we should have found $d_2 = 15.638$, which is almost exactly like 15.644, the value found (on next page) when only the first difference is used throughout.

Solving this equation of the first degree we get $x_2 = .3279$ and $\frac{d_2}{S}$ .3699 + .3927 × .054 = .3911, that is $d_2 = .3911 \times 40 = 15.644$. We also find $\frac{D_2 - d_2}{S} = .3 - .3927 \times .1 = .2607$ and $D_2 = 15.64 + .2607 \times 40 = 26.07$.

In like manner we can first find x_4 and then D_4 and d_4 from

$$\frac{d_4}{D_4} = 1 = \frac{\frac{d_4 - D_4}{S}}{\frac{D_4}{S}} = \frac{240}{100} - 1 = 1.4000,$$

or
$$\frac{.5 - x_4 \times .1}{.2571 + x_4 \times .0572} = 1.4,$$

this gives $x_4 = .7779$, $D_4 = 12.06$, $d_4 = 28.95$.

Since $\frac{D_3}{d_3} = \frac{240}{240} = 1$, we easily find $D_3 = d_3 = .5271 \times 40 = 21.08$.

The required pairs are therefore as follows:

29	26.07	21.08	12.06 on cone *A*,
12	15.64	21.08	28.95 on cone *B*.

The numerical results are the effective diameters, that is, the diameters measured to center of belt. To get the actual diameters, therefore, for this case we must subtract the thickness of belt, and if this be taken at .2″ we shall have for the actual diameters:

28.8	25.87	20.88	11.86 on cone *A*, ...
11.8	15.44	20.88	28.75 on cone *B*.

A pair of cones exactly alike and each having five steps could evidently be constructed from the first three of these pairs, as follows,

28.8	25.87	20.88	15.44	11.80
11.8	15.44	20.88	25.87	28.80

If we disregard these velocity ratios, this series can easily be modified so that the like steps shall have diameters that are an

exact number of inches; for instances by substracting 0.88 from each of the series, we get

27.92	24.99	20.00	14.56	10.92
10.92	14.56	20.00	24.99	27.92

That this series will have its own invariable length of belt is evident from Eq. 266, p. 138.

The blank spaces of the table represent impossible cases.

Special Tables when Cones are Exactly Alike

When two cone pulleys are connected by an open belt it is not only desirable that the diameters of the steps be so chosen that the belt will have the same tension on all the steps, but that the cones shall be alike, so that they may be cast from a single pattern. (Of course the velocity ratio of the two pairs of steps at the ends of the cones will be reciprocals of each other). Moreover as a matter of looks it is desirable that the differences of diameter of the adjacent steps of a cone shall be as nearly alike as the above mentioned requirements will permit. This average difference of steps is easily obtained from the data usually given in practice. For in designing a pair of equal cone pulleys the two diameters of the extreme steps are directly assumed or determined from their desired velocity ratio. The difference of these extreme diameters divided by the desired number of steps will give the average difference of the steps.

The numbers given in the table are the differences between the diameters of the adjacent steps on either cone pulley, and are accurate within half a hundredth of an inch (= .005) which in the present case is a degree of accuracy exceeding the requirements of practice.

By simply omitting a step at each end of the cone, the two tables given will be found equally well adapted for determining the diameters of cones having four and three steps respectively.

I.—Table for Finding Cone Pulley Diameters

when the two pulleys are connected by an *open* belt and are exactly *alike*. The numbers given in table are the differences between the diameters of the adjacent steps on either cone pulley, and can be employed when there are either Six or Four Steps on a Cone.

When there are *six* steps, the smallest is to be regarded as the 1st.
When there are *four* steps, the smallest is to be regarded as the 2d.

Average difference of diameters of adjacent steps.	Adjacent steps whose difference is given in table.	Distance between the Centers of Cone Pulleys.											
		10″	20″	30″	40″	50″	60″	70″	80″	90″	100″	120″	240″
1″	1st & 2d	1.13	1.06	1.04	1.03	1.02	1.02	1.02	1.02	1.01	1.01	1.01	1.00
	2d & 3d	1.06	1.03	1.02	1.02	1.01	1.01	1.01	1.01	1.01	1.01	1.00	1.00
	3d & 4th	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	4th & 5th	0.94	0.97	0.98	0.98	0.99	0.99	0.99	0.99	0.99	0.99	1.00	1.00
	5th & 6th	0.87	0.94	0.96	0.97	0.98	0.98	0.98	0.98	0.98	0.99	0.99	1.00
1½″	1st & 2d	1.79	1.64	1.60	1.57	1.56	1.55	1.54	1.54	1.53	1.53	1.52	1.51
	2d & 3d	1.64	1.57	1.55	1.54	1.53	1.52	1.54	1.52	1.51	1.51	1.51	1.51
	3d & 4th	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50
	4th & 5th	1.36	1.43	1.45	1.46	1.47	1.48	1.48	1.48	1.49	1.49	1.49	1.49
	5th & 6th	1.21	1.36	1.40	1.43	1.44	1.45	1.46	1.46	1.47	1.47	1.48	1.49
2″	1st & 2d	2.53	2.26	2.17	2.13	2.10	2.09	2.08	2.07	2.06	2.05	2.04	2.02
	2d & 3d	2.26	2.13	2.08	2.07	2.05	2.04	2.04	2.03	2.03	2.03	2.02	2.01
	3d & 4th	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
	4th & 5th	1.74	1.87	1.92	1.93	1.95	1.96	1.96	1.97	1.97	1.97	1.98	1.99
	5th & 6th	1.47	1.74	1.83	1.87	1.90	1.91	1.92	1.93	1.94	1.95	1.96	1.98
2½″	1st & 2d	3.34	2.90	2.77	2.70	2.66	2.63	2.61	2.60	2.59	2.58	2.57	2.53
	2d & 3d	2.90	2.70	2.63	2.60	2.58	2.57	2.56	2.55	2.54	2.54	2.53	2.51
	3d & 4th	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50
	4th & 5th	2.10	2.30	2.37	2.40	2.42	2.43	2.44	2.45	2.46	2.46	2.47	2.49
	5th & 6th	1.66	2.10	2.23	2.30	2.34	2.37	2.39	2.40	2.41	2.42	2.43	2.47
3″	1st & 2d	4.24	3.58	3.38	3.29	3.23	3.19	3.16	3.14	3.13	3.12	3.10	3.05
	2d & 3d	3.58	3.29	3.19	3.14	3.12	3.10	3.08	3.07	3.06	3.06	3.05	3.02
	3d & 4th	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
	4th & 5th	2.42	2.71	2.81	2.86	2.88	2.90	2.92	2.93	2.94	2.94	2.95	2.98
	5th & 6th	1.76	2.42	2.62	2.71	2.77	2.81	2.84	2.86	2.87	2.88	2.90	2.95
4″	1st & 2d		5.05	4.69	4.51	4.41	4.34	4.29	4.25	4.22	4.20	4.17	4.09
	2d & 3d	5.06	4.51	4.34	4.25	4.20	4.17	4.15	4.13	4.12	4.11	4.09	4.04
	3d & 4th	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
	4th & 5th	2.94	3.49	3.66	3.75	3.80	3.83	3.85	3.87	3.88	3.89	3.91	3.96
	5th & 6th		2.95	3.31	3.49	3.59	3.66	3.71	3.75	3.78	3.80	3.83	3.91
5″	1st & 2d		6.67	6.09	5.80	5.64	5.53	5.45	5.40	5.36	5.32	5.26	5.13
	2d & 3d	6.69	5.81	5.53	5.40	5.32	5.26	5.23	5.20	5.18	5.16	5.14	5.07
	3d & 4th	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00
	4th & 5th	3.21	4.19	4.47	4.60	4.68	4.74	4.77	4.80	4.82	4.84	4.86	4.93
	5th & 6th		3.33	3.92	4.20	4.36	4.47	4.55	4.60	4.64	4.68	4.74	4.87
6″	1st & 2d		8.48	7.58	7.17	6.92	6.77	6.66	6.58	6.51	6.45	6.38	6.20
	2d & 3d		7.17	6.77	6.58	6.46	6.38	6.33	6.29	6.25	6.23	6.19	6.10
	3d & 4th		6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00
	4th & 5th		4.83	5.23	5.42	5.54	5.62	5.67	5.71	5.75	5.77	5.81	5.90
	5th & 6th		3.52	4.42	4.83	5.08	5.23	5.34	5.42	5.49	5.55	5.62	5.80

II.—Table for Finding Cone Pulley Diameters.

when the two pulleys are connected by an *open* belt and are exactly *alike*. The numbers given in table are the differences between the diameters of the adjacent steps on either cone pulley, and can be employed when there are either Five or Three Steps on a Cone.

When there are *five* steps the smallest is to be regarded as the 1st. When there are *three* steps the smallest is to be regarded as the 2d.

Average difference of diameters of adjacent steps.	Adjacent steps whose difference is given in table.	Distance between the Centers of Cone Pulleys.											
		10″	20″	30″	40″	50″	60″	70″	80″	90″	100″	120″	240″
1″	1st & 2d	1.10	1.05	1.03	1.02	1.02	1.02	1.01	1.01	1.01	1.01	1.01	1.00
	2d & 3d	1.03	1.02	1.01	1.01	1.01	1.01	1.00	1.00	1.00	1.00	1.00	1.00
	3d & 4th	0.97	0.98	0.99	0.99	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00
	4th & 5th	0.90	0.95	0.97	0.98	0.98	0.98	0.99	0.99	0.99	0.99	0.99	1.00
1½″	1st & 2d	1.72	1.61	1.57	1.55	1.54	1.54	1.53	1.53	1.52	1.52	1.52	1.51
	2d & 3d	1.57	1.54	1.52	1.52	1.52	1.51	1.51	1.51	1.51	1.51	1.51	1.51
	3d & 4th	1.43	1.46	1.48	1.48	1.48	1.49	1.49	1.49	1.49	1.49	1.49	1.49
	4th & 5th	1.28	1.39	1.43	1.45	1.46	1.46	1.47	1.47	1.48	1.48	1.48	1.49
2″	1st & 2d	2.39	2.19	2.13	2.10	2.08	2.07	2.06	2.05	2.04	2.04	2.03	2.02
	2d & 3d	2.13	2.06	2.04	2.03	2.03	2.02	2.02	2.02	2.01	2.01	2.01	2.01
	3d & 4th	1.87	1.94	1.96	1.97	1.97	1.98	1.98	1.98	1.99	1.99	1.99	1.99
	4th & 5th	1.61	1.81	1.87	1.90	1.92	1.93	1.94	1.95	1.96	1.96	1.97	1.98
2½″	1st & 2d	3.11	2.80	2.70	2.65	2.62	2.60	2.59	2.58	2.57	2.56	2.55	2.53
	2d & 3d	2.70	2.60	2.57	2.55	2.54	2.53	2.53	2.53	2.52	2.52	2.52	2.51
	3d & 4th	2.30	2.40	2.43	2.45	2.46	2.47	2.47	2.47	2.48	2.48	2.48	2.49
	4th & 5th	1.89	2.20	2.30	2.35	2.38	2.40	2.41	2.42	2.43	2 44	2.45	2.47
3″	1st & 2d	3.90	3.43	3.29	3.22	3.17	3.14	3.13	3.11	3.10	3.09	3.07	3.04
	2d & 3d	3.29	3.14	3.10	3.07	3.06	3.05	3.04	3.04	3.03	3.03	3.02	3.01
	3d & 4th	2.71	2.86	2.90	2.93	2.94	2.95	2.96	2.96	2.97	2.97	2.98	2.99
	4th & 5th	2.10	2.57	2.71	2.78	2.83	2.86	2.87	2.89	2.90	2.91	2.93	2.96
4″	1st & 2d		4.78	4.51	4.38	4.31	4.25	4.22	4.19	4.17	4.16	4.13	4.06
	2d & 3d	4.52	4.26	4.17	4.13	4.10	4.09	4.08	4.06	4.06	4.05	4.04	4.02
	3d & 4th	3.48	3.74	3.83	3.87	3.90	3.91	3.92	3.94	3.94	3.95	3.96	3.98
	4th & 5th		3.22	3.49	3.62	3.69	3.75	3.78	3.81	3.83	3.84	3.87	3.94
5″	1st & 2d		6.23	5.80	5.60	5.48	5.40	5.34	5.29	5.27	5.24	5.20	5.10
	2d & 3d	5.81	5.40	5.27	5.20	5.16	5.13	5.11	5.10	5.09	5.08	5.07	5.04
	3d & 4th	4.19	4.60	4.73	4.80	4.84	4.87	4.89	4.90	4.91	4.92	4.93	4.96
	4th & 5th		3.77	4.20	4.40	4.52	4.60	4.66	4.71	4.73	4.76	4.80	4.90
6″	1st & 2d		7.79	7.17	6.87	6.69	6.58	6.49	6.43	6.38	6.34	6.29	6.14
	2d & 3d	7.18	6.58	6.38	6.29	6.23	6.19	6.17	6.14	6.13	6.12	6.10	6.05
	3d & 4th	4.82	5.42	5.62	5.71	5.77	5.81	5.83	5.86	5.87	5.88	5.90	5.95
	4th & 5th		4.21	4.83	5.13	5.31	5.42	5.51	5.57	5.62	5.66	5.71	5.86

We will now illustrate the use of the tables by a few examples.

EXAMPLE 1.—Suppose the centers of the pulley shafts to be 60″ apart, and that the average difference between the adjacent steps is to be $2\frac{1}{2}$″; also that each cone is to have six steps, the smallest having a diameter equal to 5″.

To find the remaining diameters we look in Table I (corresponding to cone pulleys with six steps) under 60″ and opposite $2\frac{1}{2}$″ and obtain the differences.

2.63 2.57 2.50 2.43 2.37

Each of these differences is added to the smaller diameter of two adjacent steps to which it corresponds, thus:

5.	7.63	10.20	12.70	15.13
2.63	2.57	2.50	2.43	2.37
7.63	10.20	12.70	15.13	17.50

These sums represent the actual diameters of the second, third, fourth, fifth and sixth steps respectively.

EXAMPLE 2.—If we suppose the same conditions as in Example 1, with the exception that each cone is to have four steps instead of six, we obtain the required diameters by omitting the end differences of the above example, and then adding the remaining differences as follows:

5.	7.57	10.07
2.57	2.50	2.43
7.57	10.07	12.50

These sums represent the actual diameters of 2d, 3d, and 4th steps respectively. If the smallest diameter had not been assumed equal to 5″ we might have dropped a step at each end of the six-stepped cone of the preceding example, and employed the remaining four diameters—7.63, 9.80, 12.30, 15.13—for one four-step cone.

The preceding examples show that we can assume the size of the smallest step anything that we please and, other things being equal, can make the required cones large or mall.

EXAMPLE 3.—Let distance apart of the centers = 30″, the average difference between adjacent steps = 2″, the smallest steps have a diameter = 4″, and the number of steps on each cone pulley = 5, then from Table II, under 30″ and opposite 2″ we obtain the differences,

2.13 2.04 1.96 1.87

and then adding as before we get the required diameters,

6.13 8.17 10.13 12.00

EXAMPLE 4.—Let the conditions be as in the preceding example, the cone pulley having, however, three steps instead of five, then dropping the end differences and adding,

4.	6.04
2.04	1.96
6.04	8.00

we get 4, 6.04 and 8 as the diameters of the cone.

EXAMPLE 5.—Let the distance apart of the centers be 60″, the average distance between the adjacent steps be $2\frac{1}{8}$″, the smallest step 7″ and number of steps = 5. Now an inspection of Table II will show that it contains no horizontal lines corresponding to the average difference $2\frac{1}{8}$″, we cannot, therefore, as heretofore, obtain the required differences directly, but must interpolate as follows: Since $2\frac{1}{8}$″ is quarter way between 2″ and $2\frac{1}{2}$″, the numbers corresponding to $2\frac{1}{8}$″ (for any given distance apart of the centers) will be quarter way between the numbers of the table corresponding to 2″ and $2\frac{1}{2}$″.

Thus we have, in Table II, under 60″ and

opposite $2\frac{1}{2}$″	2.40	2.47	2.53	2.60
opposite 2″	1.93	1.98	2.02	2.07
	.47	.49	.51	.53

Dividing these differences by 4, we get

	.12	.12	.13	.13
to which we add	1.93	1.98	2.02	2.98

and get for the differences corresponding to $2\frac{1}{8}$,

	2.20	2.15	2.10	2.05
adding as before	7.	9.20	11.35	13.45
the diams. will be	9.20″	11.35″	13.45″	15.50″

When we assume a given velocity ratio with the belt at one end of the cones, and also assume the diameter of the smallest step, we can easily obtain the largest step (increased by thickness of belt) by multiplying the smallest one (increased by thickness of belt) by the assumed velocity ratio; the difference between the largest and smallest steps, divided by the number of steps less one, will give the average difference between the steps; we can then proceed as in the above examples.

Construct diagram on cross-section paper by means of the coordinates given on Plate XII. Solve the problems 1, 2, and 3 given on pages 142, 145 and 147 by means of the diagram; also solve the same problems for case of crossed belt. Take in addition the following examples:

Example A.—The driving shaft makes 240 revolutions per minute, distance of the axes $S = 40''$. The velocity ratios transmitted by the 5 steps of the cone vary as the terms of a geometrical series, whose first term $= 4$; the two cones must be alike so that they can be cast from the same pattern. Assume smallest step $= 5''$.

Find the actual diameters of the five steps on each cone by means of both table and diagram, when thickness of belt $= \frac{7}{32}''$.

Example B.—Driving shaft makes 200 revolutions per minute; distance of axes $= 54''$; velocity ratio transmitted by first pair of steps $= 4$; the two cones are alike and have each 4 steps whose adjacent diameters differ from one another by the average

difference 6″. Thickness of belt $= \frac{3}{16}$″. Find actual diameters on both cones, and revolutions per minute of driven shaft.

EXAMPLE C.—Diameters on cone $B =$ 8 12 16 20
Diameters on cone $A =$ 18 — — —

Distance between axes $= S =$ 125″. Find the unknown diameters by means of table; also calculate for crossed belt.

The answers to the following problems may be found in Trans. Amer. Soc. M. E., Vol. X, pp. 291-293.

EXAMPLE D.—*Given*, distance between axes = 32″ and

velocity ratio	1	1.25	1.50	1.75	2.	2.5	3	4	5	6	7
Diam. on cone B = 24″	—	—	—	—	—	—	—	—	—	—	—
" " " A = 24″	—	—	—	—	—	—	—	—	—	—	—

Find the unknown diameters by means of table.

EXAMPLE E.—*Given*, distance between axes = 100″ and

velocity ratio =	3	2	1
Diam. on cone B =	24	—	—
" " " A =	8	—	—

Find the unknown diameters by means of table.

EXAMPLE F.—Given, distance between axes = 41.625″ and

velocity ratio =	$2\frac{2}{3}$	1.75	1.20
Diam. on cone B =	12	—	—
" " " A =	4.5	—	—

Find the unknown diameters by means of table.

EXAMPLE G.—*Given*, distance between the axes = 13.5″ and

velocity ratio =	6.5	3.1667	1.9318	1.2982	1.1129	1.6531
Diam. on cone B =	13	—	—	—	—	—
" " " A =	2	—	—	—	—	—

Find the unknown diameters by means of the table.

PLATE XIII.

ROTATING PIECES.

a = constant determined by experience.

A = lever arm of the bending force.

b and b' = constants depending on a and f.

b_1 = breadth of ring for collar journal (measured on radius of ring).

c and c' = constants depending on a and f.

d = diameter of journal shaft or pivot; also diameter of necked portion of shaft.

d_0 = diameter of shaft before it is necked or reduced.

d' = mean diameter of rings for collar journal.

d_1 = diameter of shaft when speed has been increased to n_1 revolutions per minute.

E = modulus of elasticity of the material.

f = permissible working stress of material (in pounds per □″).

f_b = permissible working stress of material (in pounds per □″) when shaft is subject to bending.

f_t = ditto—when shaft is subject to twisting.

G = modulus of shearing, or transverse elasticity of material, $= \frac{2}{5} E$.

h = thickness of inner edge of ring for collar journal.

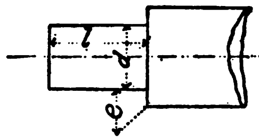
h_1 = thickness of outer edge of ring for collar journal.

H = number of horse-powers transmitted.

l = length of journal or bearing, in inches.

L = total length of shafting, etc., in feet.

L_m = mean length of shafting, etc., in feet.



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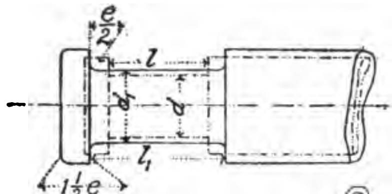

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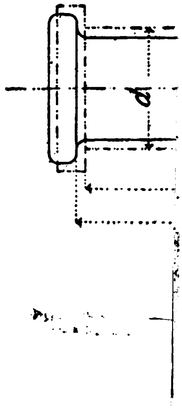

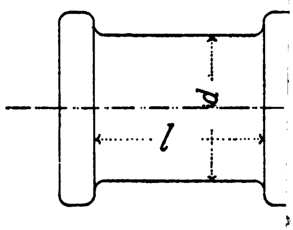

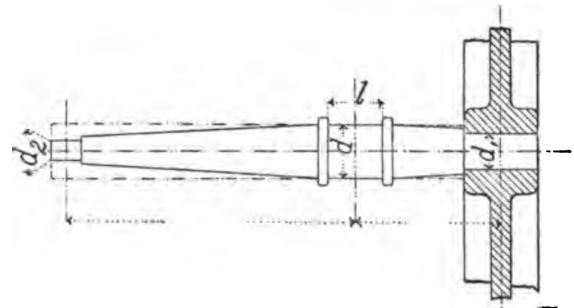

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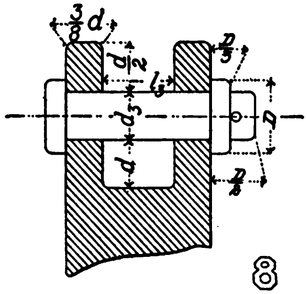

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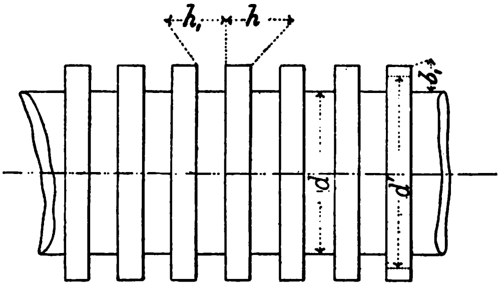

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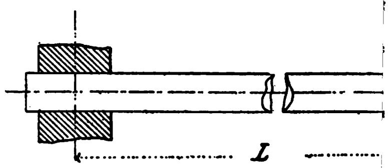


ROTATING PIECE

XIII

L = distance between adjacent supports or bearings, in inches.

L_1 = distance between adjacent supports or bearings, in feet.

L_o = length of hollow portion of shafting.

M = resultant or equivalent of all bending and twisting moments.

M_{1b} = bending moment of a single force in inch pounds.

M_{2b} = bending moment of a single force in inch pounds.

M_b = bending moment in inch pounds.

M_t = twisting moment in inch pounds.

$(M_b)_i$ = ideal or equivalent bending moment.

$(M_t)_i$ = ideal or equivalent twisting moment.

n = revolutions or oscillations per minute of journal or pivot.

p = permissible presssure per □″ of projected area of rubbing surface (*i. e.*, per □″ of axial section) when journal is new.

p_o = ditto—when journal and bearings are somewhat worn.

P = load to which journal or pivot is subjected, in pounds.

PR = twisting moment on shafting or shaft in inch-pounds.

Q = bending load (supposed to be acting midway between bearings) in pounds.

R = lever arm of the twisting force.

U = work absorbed in foot-pounds.

V = volume of shaft in cubic inches.

W = work absorbed by a shaft during a twist.

w = total weight of shafting.

z = number of collars for thrust journals.

α $= \dfrac{S}{L} = \dfrac{\text{deflection at center of shaft in inches}}{\text{distance between bearings in inches}}.$

β° = allowable twist in degrees for length L.

γ = angle included between the forces whose bending moments are M_{1b} and M_{2b}.

The force which acts on a journal may be either

Constant in Direction or Variable in Direction.

In the latter case the force usually acts alternately on the two sides of the journal, and while it is unfavorable as regards the strength of the piece, it is very favorable as regards the efficiency of the lubrication; the partial vacum (and perhaps the capillary action) on the relieved side of the journal draws in the oil, thus greatly increasing the efficiency of the lubrication, and permitting a higher pressure per □″ between the rubbing surfaces than is allowable on those journals in which, the direction of force being constant, there is a constant effort to force the lubricant from between the rubbing surfaces.

In order that account may be taken of these conditions and also that the wear may not become excessive at high speeds, the assumption is made, that

$$p = \frac{a}{n}. \qquad (272)$$

In order that the important condition of sufficient strength be not neglected, we must make use of the following formulas into which given values of p have been introduced.

FOR END JOURNALS.

$$\frac{l}{d} = .471\sqrt{\frac{f}{p}} = b\sqrt{n}, \qquad (273)$$

$$d = 2.26\sqrt{\frac{l}{d}\frac{P}{f}} = c\sqrt{P}\sqrt[4]{n}. \qquad (274)$$

FOR JOURNAL FIXED TO FORKED END OF LEVERS OR RODS.

$$\frac{l}{d} = .943\sqrt{\frac{f}{p}} = b'\sqrt{n}, \qquad (275)$$

$$d = 1.13\sqrt{\frac{l}{d}\frac{P}{f}} = c'\sqrt{P}\sqrt[4]{n}. \qquad (276)$$

In neck journals subjected to bending and twisting moments, the diameter can be obtained from the general formula for strength,

$$d = 2.167\sqrt[3]{\frac{M}{f}}, \qquad (277)$$

where M represnts the resultant or equivalent of all the bending and twisting moments to which the neck journal is subjected. This equivalent moment is also called the ideal bending moment.

The length of a neck journal can be made equal to that of an end journal subjected to the same load and having the same number of revolutions per minute. In the case of shafting the neck journals may be increased in length indefinitely without diminishing their strength. Generally formulas for collar journals and pivots do not need to take strength into account, because the crushing strength per $\square''$ is greatly in excess of the permissible pressure p between the rubbing surfaces. It is only in pivots subjected to great loads which move very slowly and intermittently—for example, such as are used in turn-tables—that strength must be taken into account. This is done by making $p =$ or $< f$.

The dimensions of journals, etc., obtained in this manner presuppose *Good Continuous Lubrication and Good Workmanship in the Fitting and Setting up of the Journals and their Boxes.*

These dimensions should be regarded as the minimum values consistent with strength and a desirable durability; we do not mean by this that, if the journal length is taken smaller than given by the formulas, the journal will necessarily heat and be useless, but that undue wear of both journal and bearings is liable to occur. Whenever such considerations as the want of space, the reduction of the lever arms of bending moments, etc., require that the journal's length be shorter than given by these formulas, we must increase the diameter to an amount that will keep the permissible pressure p below the maximum given in the table; this increase in diameter will increase the loss of work due to friction, but will increase durability.

FORMULAS FOR END JOURNALS.

		Direction of Load Constant.			Direction of Load Variable.			
		Wro't Iron.	*Casi Iron.*	*Steel.*	*Wro't Iron.*	*Cast Iron.*	*Steel.*	
Turning very slowly and intermittently.	$p_0 =$	8500	4250	14200	8500	4250	14200	
	$f =$	8500	4250	14200	8500	4250	14200	
	$\frac{l}{d} =$	0.5	0 5	0.5	0.5	0.5	0.5	
	$d =$	$.0173\sqrt{P}$	$.0245\sqrt{P}$	$.0133\sqrt{P}$	$.0173\sqrt{P}$	$.0245\sqrt{P}$	$.0133\sqrt{P}$.	(278)
$n =$ or < 150	$p =$	700	350	700	1400	700	1400	
	$f =$	8500	4250	14200	7100	3600	11800	
	$\frac{l}{d} =$	1.5	1.5	1.94	1.	1.	1.3	
	$d =$	$.031\sqrt{P}$	$.043\sqrt{P}$	$.0266\sqrt{P}$	$.0266\sqrt{P}$	$.037\sqrt{P}$	$.0234\sqrt{P}$	(279)
$n =$ or > 150	$a =$	107000	—	107000	214000	—	214000	
	$f =$	8500	—	14200	7100	—	11800	
	$\frac{l}{d} =$	$.13\sqrt{n}$	—	$.17\sqrt{n}$	$.08\sqrt{n}$	—	$.10\sqrt{n}$	
	$d =$	$.0237\sqrt{\frac{l}{d}}\sqrt{P}$	—	$.019\sqrt{\frac{l}{d}}\sqrt{P}$	$.027\sqrt{\frac{l}{d}}\sqrt{P}$	—	$.021\sqrt{\frac{l}{d}}\sqrt{P}$.	(280)

VALUES OF P IN POUNDS WHEN n = OR < 150.

d = Diameter of Journal.	Unit for Brasses = e.	Direction of Load Constant. Wrought Iron. $\frac{l}{d} = 1.5$	Direction of Load Constant. Cast Iron. $\frac{l}{d} = 1.5$	Direction of Load Constant. Steel. $\frac{l}{d} = 1.94$	Direction of Load Variable. Wrought Iron. $\frac{l}{d} = 1$	Direction of Load Variable. Cast Iron. $\frac{l}{d} = 1$	Direction of Load Variable. Steel. $\frac{l}{d} = 1.3$
1.0	0,20	1124	555	1422	1422	724	1836
1.2	0.20	1639	799	2048	2048	1024	2644
1.4	0.25	2203	1090	2788	2788	1395	3600
1.6	0.25	2912	1422	3641	3641	1820	4702
1.8	0.25	3684	1800	4608	4608	2305	5951
2.0	0.28	4552	2222	5689	5689	2845	7346
2.2	0.28	5507	2684	6884	6884	3443	8889
2.4	0.32	6554	3200	8193	8193	4096	10580
2.6	0.32	7692	3755	9615	9615	4809	12416
2.8	0.32	8921	4356	11151	11151	5576	14399
3.0	0.32	10241	5006	12801	12801	6402	16531
3.2	0.36	11651	5689	14565	14565	7282	18807
3.4	0.40	13154	6422	16442	16442		21232
3.6	0.40	14747	7201	18434	18434		23803
3.8	0.40	16431	8022	20539	20539		26021
4.0	0.40	18206	8889	22757	22757		29387
4.2	0.40	20072	9802	25090	25090		32400
4.4	0.44	22029	10757	27410	27410		35558
4.6	0.44	24078	11756	30097	30097		38865
4.8	0.48	26216	12801	32770	32770		42317
5.2	0.48	30708	15025	38460	38460		49663
5.6	0.52	35684	17423	44627	44627		57599
6.0	0.52	40963	20001	51204	51204		66120
6.4	0.60	46607	22757	58259	58259		75231
6.8	0.60	52615	25691	65769	65769		84928
7.2	0.64	58988	28802	73734	73734		95212
7.6	0.64	65723	32092	82154	82154		106090
8.0	0.68	72823	25558	91029	91029		117550
8.4	0.72	80289	39201	100360	100360		129600
8.8	0.72	88117	43025	110150	110150		142230
9.2	0.76	95126	47025	120390	120390		155450
9.6	0.80	104865	51204	131080	131080		169270
10.4	0.84	123070	60093	153840	153840		198650
11.2	0.92	142720	69695	178420	178420		230600
12.0	0 96	163850	80005	204801	204801		264480

FORMULAS FOR JOURNALS (PINS) FIXED IN THE FORKED ENDS OF LEVERS OR RODS.

		Direction of Load Constant.			Direction of Load Variable.			
		Wro't Iron.	*Cast Iron.*	*Steel.*	*Wro't Iron.*	*Cast Iron.*	*Steel.*	
Turning very slowly and intermittently.	$p_0 =$	8500	4250	14200	8500	4250	14200	
	$f =$	8500	4250	14200	8500	4250	14200	
	$\frac{l}{d} =$	1	1	1	1	1	1	
	$d =$	$.0122\sqrt{P}$	$.0173\sqrt{P}$	$.0096\sqrt{P}$	$.0122\sqrt{P}$	$.0173\sqrt{P}$	$.0096\sqrt{P}$	(281)
$n =$ or < 150	$p =$	700	350	700	1400	700	1400	
	$f =$	8500	4250	14200	7100	3550	11800	
	$\frac{l}{d} =$	3	3	4	2	2	5	
	$d =$	$.0213\sqrt{P}$	$.03\sqrt{P}$	$.0186\sqrt{P}$	$.0186\sqrt{P}$	$.0266\sqrt{P}$	$.016\sqrt{P}$	(282)

n rarely exceeds 150 except in the connecting rods of high speed engines, and then the average speed of the rubbing surfaces at the cross-head pin is only $\frac{1}{16}$th of that at the crank-pin. This fact permits the pin at the cross-head to be made shorter than the above formulas call for, but care should be taken that $\frac{P}{ld} =$ or < 1400.

Formula for Flat Pivots.

		Wrought Iron and Steel on Gun Metal.	*Cast Iron on Gun Metal.*	*Iron or Steel on Lignum Vitæ immersed in, or well moistened by, water.*	
Turning very slowly and intermittently.	$p =$	8500	4250	——	
	$d =$	$0.035\sqrt{P}$	$0.05\sqrt{P}$	——	(283)
$n =$ or < 150	$p =$	700	350	1400	
	$d =$	$0.05\sqrt{P}$	$0.07\sqrt{P}$	$0.035\sqrt{P}$	(284)
$n =$ or > 150	$a =$	107000	——	$p = 1400$	
	$d =$	$0.004\sqrt{Pn}$	——	$d = 0.035\sqrt{P}$.	(285)

Flat Pivots.

Values of P iu Pounds.

$d=$	$0.035\sqrt{P}$	$0.050\sqrt{P}$	$0.070\sqrt{P}$	$d=$	$0.035\sqrt{P}$	$0.05\sqrt{P}$	$0.07\sqrt{I}$
0.60	298.12	147.92	75.098	4.8	19096.	9503.4	4772.2
0.80	531.25	263.99	131.99	5.2	22412.	11153.	5602.9
1.00	828.46	411.91	205.20	5.6	25991.	12935.	6497.3
1.2	1192.5	593.96	298.12	6.0	29837.	14849.	7462.
1.4	1624.9	805.60	405.08	6.4	33949.	16895.	8486.1
1.6	2121.0	1056.0	530.25	6.8	38326.	19073.	9580.8
1.8	2685.4	1335.9	671.34	7.2	42966.	21383.	10742.
2.0	3315.7	1649.9	828.36	7.6	47883.	23825.	11878.
2.2	4012.1	1995.8	1001.3	8.0	53044.	26299.	13261.
2.4	4772.2	2375.9	1192.5	8.4	58482.	29084.	14620.
2.6	5602.9	2787.7	1399.7	8.8	64184.	31942	16046.
2.8	6497.2	3233.8	1624.9	9.2	70152.	34912.	17537.
3.0	7462.1	3711.7	1863.8	9.6	76385.	38014.	19096.
3.2	8486.0	4223.8	2121.0	10.0	82882.	41248.	20720.
3.4	9580.8	4767.6	2396.3	10.4	89646.	44612.	22411.
3.6	10742.	5345.6	2685.4	10.8	96672.	48112.	24168.
3.8	11968.	5955.5	2992.6	11.2	103890.	51740.	25991.
4.0	13261.	6599.6	3315.7	11.6	111530.	55502.	27882.
4.2	14620.	7275.5	3654.9	12.0	119350.	59396.	29837.
4 4	16046.	7985.5	4012.1				

FORMULAS FOR COLLAR JOURNALS OF SMALL UPRIGHT SHAFTS.

$$d' = 0.01 \sqrt[3]{\frac{P^2 n^2}{z^2}}, \qquad (286)$$

b_1 varies from $\frac{1}{6}d'$ to $\frac{1}{10}d'$, $b_1 = \frac{1}{6}d'$ is used for small shafts, or

$$b_1 = 0.24\sqrt{d'}, \qquad (287)$$

h and h_1 often equal b, sometimes $h_1 < h$.

$$p = \frac{a}{n} = \frac{45000}{n}, \qquad (288)$$

$$P = z\pi d' b p. \qquad (289)$$

Revolutions per minute = n =	150	400	450	600	1000
p =	300	150	100	75	45

For large screw-propeller shafts, see *Reuleaux'* table of executed designs in Konstrukteur.

FORMULAS FOR SHAFTS.

TORSION.

	Wrought Iron.	Steel.	
Strength.	$\beta^\circ = \text{or} < \frac{3}{40}L$ and $d = \text{or} > 11.0$,	$\beta^\circ = \text{or} < \frac{3}{40}L$ and $d = \text{or} > 16$	
	$f_t = 6800$,	$f_t = 11400$.	
	$d = 0.091\sqrt[3]{PR} = 3.63\sqrt[3]{\frac{H}{n}}$,	$d = 0.08\sqrt[3]{PR} = 3.2\sqrt[3]{\frac{H}{n}}$.	(290)
Stiffness.	$\beta^\circ = \frac{3}{40}L$ and $d = \text{or} < 11.0$,	$\beta^\circ = \frac{3}{40}L$ and $d = \text{or} < 16$.	
	$G = 10800000$,	$G = 10800000$.	
	$d = 0.3\sqrt[4]{PR} = 4.75\sqrt[4]{\frac{H}{n}}$,	$d = 0.3\sqrt[4]{PR} = 4.75\sqrt[4]{\frac{H}{n}}$.	(291)

Use table on page 86.

No allowance in these formulas or table for sunk keys.

FLEXURE.

	Wrought Iron.	Steel.	
Strength.	a = or $<$ 0.001 and $\frac{L}{d}$ = or $<$ 20,	a = or $<$ 0.001 and $\frac{L}{d}$ = or $<$ 12.	
	$f_b = 8500$,	$f_b = 14200$.	
	$L = 3350\frac{d^3}{Q}$,	$L = 5590\frac{d^3}{Q}$.	(292)
Stiffness.	a = 0.001 and $\frac{L}{d}$ = or $>$ 20,	a = 0.001 and $\frac{L}{d}$ = or $>$ 13.	
	$G = 10800000$,	$G = 10800000$.	
	$L = 260\frac{d^2}{\sqrt{Q}}$,	$L = 260\frac{d^2}{\sqrt{Q}}$.	(293)

Obtained by supposing the shaft to be supported on its bearings, and the whole load (weight of shaft pulleys, pull of belts) concentrated at the middle. Deflection with uniformly distributed load $\frac{3}{5}$ that with load at middle.

Other things being equal, we can say that the deflection for beam fixed at both ends is $\frac{1}{4}$ to $\frac{1}{6}$ that which obtains when the

ends are supported; probably the case of shafting in practice approximates more closely to a beam with its ends *supported* than when they are fixed, at any rate we are on the safe side in assuming them *supported.*

Rankine assumed for shafting $\alpha = \frac{1}{2000} = 0.0005$, shaft supported at both ends, but total load ($= \frac{4}{3}$ weight of shaft) *uniformly* distributed, which gave the following formula for wrought iron and steel shaftings:

$$L_1 = 4.5\sqrt[3]{d^2}, \tag{294}$$

hence the table—

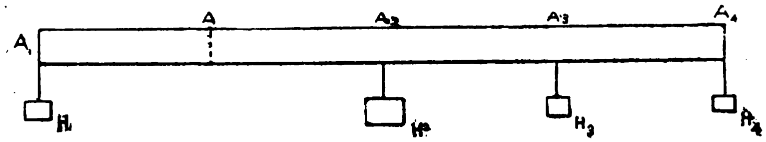
Diameter in inches	1″	2″	2½″	3½″	4″
Length in feet	4.5	7.16	8.38	9.36	11.34

According to *Reuleaux* the twist may be kept within moderate limits by employing the formulas,

$$\beta^\circ = 0.195\sqrt{L_m}, \tag{295}$$

$$d = 3.75\sqrt{\frac{H}{n}\sqrt[3]{L_m}}. \tag{296}$$

Here L_m has the following signification:



If we suppose the shaft to receive power at A and to deliver portions of it H_1, H_2, H_3, etc., at A_1, A_2, A_3, respectively, we can take account of the twist due to such distribution of power by supposing the delivered quantities of work to represnt weights suspended from the shaft at the points at which work is delivered. Then the distance of the center of gravity of these

weights from the point at which power is received will be the value L_m to be introduced in the formula, that is,

$$L_m = \frac{H_2 \times AA_2 + H_3 \times AA_3 + H_4 \times AA_4, \text{ etc.}}{H_2 + H_3 + H_4, \text{ etc.}} \tag{297}$$

The relation between shocks or sudden twisting resistances and the dimensions of a shaft, is given by the following formulas:

For solid shaft,

$$W = \frac{(f_t)^2}{4G} V, \tag{298}$$

and for hollow shaft,

$$W = \frac{\pi}{16} \frac{f_t^2}{G} \frac{d^4 - d_o^4}{d^2} L_o, \tag{299}$$

L_o being the length of the hollow portion of the shafting.

To ascertain the influence that necking a shaft has on its capacity to resist shock, see *Weisbach-Herrmann's* Machinery of Transmission, p. 60.

In the same work, on p. 67, will be found a discussion of the effect of speed on the work of friction, and on the diameter or weight of the shaft. We there find, the work of friction, the loss of power and consumption of lubricants, will generally be the smaller, the greater the increase of speed. Since an increase of speed is accompanied by other advantages—namely, lighter and less costly wheels, pulleys, bearings and transmission pieces, by smaller pitches for the teeth, by narrower belts, etc.—it is customary to run shafting at speeds above a certain limit. What this limit is in various cases at the present time will appear in the following extract from *Wm. Sellers & Co.'s* "MACHINE TOOLS."

"The tendency now is to increase, rather than diminish, the speed of the line shafts, and good practice is to run shafts for machine-shop purposes at 120 revolutions, for wood-working machinery at 250 revolutions, and for cotton and woolen mills at from 300 to 400 revolutions per minute. Hollow, or pipe, shafting has been made to run at 600 revolutions per minute very

satisfactorily. This kind of shafting is, however, too costly to be generally introduced. *Mr. James B. Francis,* of Lowell, says that since the decrease of water-power in that town, or rather the rapid increase in the factories, they have been obliged to economize their power, and they are doing so by using smaller shafts at higher velocities; they have even made extended lines only $1\frac{1}{4}''$ in diameter. They so arrange the mill as to secure a hanger close to each transmitting pulley. The torsion in long lines limits the smallness of the shaft used, and in all probability the best result will be found to be obtained in the use of not less than $1\frac{3}{4}''$ diameter for the smallest line shafts in cotton mills.

When very long lines of shafting are constructed of small, or comparatively small, diameter, such lines are liable to some irregularities of speed, owing to the torsion or twisting of the shaft as power is taken from them in more or less irregular manner.

Shafts driving looms may at one time be under the strain of driving all the looms belted from them, but as some looms are stopped the strain on the shaft becomes relaxed, and the torsional strain drives some part of the line ahead, and again retards it when the looms are started up. This irregularity is in some cases a matter of serious consideration, as in the instance of driving weaving-machinery. The looms are provided with delicate stop motion, whereby the breaking of a thread knocks off the belt-shifter and stops the loom. An irregular driving motion is apt to cause the looms to knock off, as it is called, hence the stopping of one or more may cause others near them to stop also. This may in a measure be arrested by providing fly wheels at intervals on the shaft, so heavy at their rim as to act as a constant retardant and storer of power, which power is given back upon any reaction on the shaft, and thus the strain is equalized. We mention this as, at the present time, it is occupying the thoughts of prominent mill-wrights, and the relative advantage and disadvantage of light and heavy shafts, is being discussed, and is influencing the practice of modern mill construction.

There are now running in some factories, lines of shafting 1000 feet long each. The power is generally applied to the shaft in the center of the mill, and the line extended each way from this. The head shaft being, say 5″ in diameter, the shafts extending each way are made smaller in proportiion to the rate of distribution, so that from 5″ they often taper down to $1\frac{3}{4}$″. In coupling shafts of different sizes it is customary to reduce the end of the larger one to the size of the smaller shaft and then to use a coupling suited to the smaller size. The rapidity with which the reduction of the size of the sections is made, must depend in all cases upon the distribution of power. For instance, if a line of any length whatever receives its power at one end, and transmits the same amount of power at its other end, such shaft must be of uniform diameter; but if it distributes its power at regular intervals along its length, the shaft may be made in sections of a size proportioned to the power given off.

To determine the size of shaft for the transmission of a given power, a safe formula is,

$$d = 5\sqrt[3]{\frac{H}{n}}. \qquad (300)$$

This gives a shaft strong enough to resist flexure, if the bearings are not too far apart. The distance apart that the bearings should be placed is an important consideration. Modern millwrights differ slightly in opinion in this respect; some construct their mills with beams *9 feet 6 inches* apart, and with but one hanger under each beam, others say *8 feet* apart gives a better result. We are clearly of the opinion that with *8 feet* distance, and with a shafting lighter in proportion, the best result is obtained.

All turned shafts are made from merchantable sizes of round bar iron, and in turning $\frac{1}{16}$″ is taken off in diameter, so that what is called a 2″ shaft is really only $1\frac{15}{16}$″ in diameter, and so of other sizes—they are all $\frac{1}{16}$″ less than their name implies, and the couplings, hangers, etc., are made to conform to these sizes."

LENGTH OF BEARINGS FOR SHAFTING.

Wm. Sellers & Co.

Nominal size of shaft,

$1\frac{3}{4}''$ $2''$ $2\frac{1}{4}''$ $2\frac{1}{2}''$ $2\frac{3}{4}''$ $3''$ $3\frac{1}{4}''$ $3\frac{1}{2}''$ $4''$ $4\frac{1}{2}''$ $5''$ $5\frac{1}{2}''$ $6''$.

Length of bearings,

$7''$ $8''$ $9''$ $10''$ $11''$ $12''$ $13''$ $14''$ $16''$ $18''$ $20''$ $22''$ $24''$.

Fig. .— *End Journal of Wrought Iron on Cast Iron Bearing.*

This form of journal is used in the cheaper kinds of work and where speeds $n =$ or < 150 are employed. Take

$$d = .04\sqrt{P} \text{ and } \frac{l}{d} = 1.4. \tag{301}$$

$$c = .07d + \tfrac{1}{8}''. \tag{302}$$

Assume $P = 550$ pounds. $n = 100$.

Fig. 2.—*End Journal of Wrought Iron on Gun or Babbitt Metal Bearings.*

The shoulders and rounded corners here employed are characteristic of the better class of journals.

Assume $P = 400$, $n = 150$. Draw in red lines.

The resulting dimensions d and l obtained from these data will be smaller than are generally employed, though with these dimensions the journal would be stiff, strong and durable enough for the load and speed to which it is subjected; if, however, as a matter of taste, a greater length l_1 be thought desirable, and we wish to find the diameter d_1 of a journal which, with the new length l_1, will give the same strength as in the first calculated journal, we must make use of the following formula:

$$\frac{l_1}{l} = \left(\frac{d_1}{d}\right)^3. \tag{303}$$

Assume $l_1 = 2l$, calculate d_1 and draw in black lines.

Fig. 3.—*Steel Journal on Gun or Babbitt Metal Bearings.*

A steel journal, on account of its greater strength, can be of smaller diameter for a given length. The smaller the diameter, the smaller will be the work of friction; diminution of diameter is, however, limited by the conditions of strength and stiffness.

Calculate the dimensions of the journal of a *car axle* running under the following conditions: Speed of train = 40 miles per hour; truck wheel diameter = 2.75 feet; load on each journal of axle = 10000 pounds.

Since the pressure of the journal is frequently relieved by the rising of the car in its springs whilst running, and since the axle is subjected to shocks from the same cause, we can assume a greater permissible pressure between the rubbing surfaces than for ordinary journals for which the load remains constant, but the working stress per □'' must be greater. The conditions as regards lubrication and strength are therefore very similar to the case in which the direction of the load varies, and the dimensions may therefore be calculated from the formulas given for that case. In standard car axle journal $l = 7''$, $d = 3\frac{3}{4}''$. Draw in red lines.

Fig. 4.—*Hollow Cast Iron Journal on Gun Metal Bearings.*

Assume $P = 16900$ pounds. $n = 50$.

Draw in red lines the solid journal having the same strength, the same length and subjected to the same load.

Assume $\frac{d_1}{d_2} = 0.6$, then will

$$d = \frac{d}{\sqrt[3]{1 - \left(\frac{d_1}{d_2}\right)^4}}. \qquad (304)$$

Draw hollow journal in **black lines.**

Fig. 5.—*Wrought Iron Axle with Neck Journal on Gun Metal Bearings.*

Assume dimensions as in figure, and weight of pulley or wheel $= 8$ tons, $n = 150$. The following formula will serve for most cases that arise:

$$d = 2.167\sqrt[3]{\frac{M}{f}}, \tag{305}$$

when $M =$ bending moment in inch-pounds.

When the axle (as the crank shaft of the steam engine) is subject to both a bending moment M_b and a twisting moment M_t we can find an *ideal* bending moment M_i from the formula,

$$M_i = \tfrac{3}{8}M_b + \tfrac{5}{8}\sqrt{M_b^2 + M_t^2}, \tag{306}$$

then will

$$d = 2.167\sqrt[3]{\frac{M_i}{f}}. \tag{307}$$

If the axle is subjected to several bending moments we must consider M_b (in the above formula for M_i) as the *resultant* bending moment.

Calculate the length of neck journal as if it were an ordinary end journal, then calculate the diameter of the neck journal by means of the formulas just given.

Calculate also the dimensions of the end journal of the axle. Usually axles and shafts are not tapered as shown in figure; it costs less to make them of uniform thickness, though the loss of work due to friction on end journal is greatly increased.

Fig. 6.—*Steel Crank Pin on Gun Metal Bearings.*

Assume diameter of cylinder of steam engine to which crank pin belongs $= 18''$, and the initial pressure of steam $= 70$ pounds.

Fig. 7.—*Knuckle Joint.*

The proportions given are empirical but suitable for small work. Assume $d = 1\frac{1}{4}''$.

The diameter of pin (theoretically) need not exceed 0.7.

Fig. 8.—*Joint Pin of Steel on Gun Metal Bearing.*

Assume $P = 900$, $n = 75$. Calculate d and l of equivalent end journal. Make $l_3 = l$, then $d_3 = \frac{5}{8}d$.

When forces are considerable and there is space enough, the pin should be proportionate according to formulas for journals fixed to forked levers or rods. The method just given (making $l_3 = l$, etc.) may, however, be employed for cross-head pins, care being taken that,

$$D = 1\tfrac{1}{2}d_3 + \tfrac{1}{16}$$

$$\frac{D}{l_3 d_3} = \text{or} < 1400. \qquad (308)$$

Fig. 9.—*Collar Journal.*

This form of journal is much used in the shafts of screw-propellers, and whenever it is not desirable to take the thrust of the shaft (in the direction of its axis) by a pivot placed at its end. Let d' = mean diameter of collars.

Assume $n = 500$, $P = 1200$, $d' = 2.25$, $h = \frac{5}{8}b_1$, $h_1 = b_1$.

Fig. 10.—*Shafting, Shafts and Axles.*

The first term is applied to the long rotating pieces which drive the machinery of the shop; the second to shorter pieces than the first, but, like them, principally subjected to twisting stresses; for example, we speak of engine shafts and propeller shafts. The third term is usually applied to rotating pieces which are principally subjected to bending stresses, as the axle of a water wheel.

Length of bearing on journal may be from $1.5d$ to $4d$, the greater the length, the greater the durability.

Assume shafting to be 250 feet long and that the work is given off uniformly along this length, 40 horse-power being the total amount of work transmitted at 130 revolutions per minute.

From a *Seller's Shearing Machine* we get the following data: the shaft driving the eccentric (which reciprocates the slide carrying the shears) is 54″ long and makes 12 revolutions per minute (*i. e.*, the shears make 12 cuts per minute). The machine can cut a plate $\frac{5}{8}''$ thick and $21\frac{1}{2}''$ long. Supposing the fly wheel on the pulley shaft sufficiently heavy to store up the energy necessary to make the cut, calculate the diameter of the shaft which drives the slides, on each of the three suppositions:

(*a*) The stiffness of the shaft is to be such that the angle of torsion $\beta^\circ = \frac{3}{40}L$. ($L$ in feet.)

(*b*) The shaft is continually subjected to stress, the time during which the shears are cutting being taken at $\frac{1}{4}$ the period during which the shears make one up and down motion; also angle of twist $\beta^\circ = < \frac{3}{40}L$.

(*c*) The shaft must be capable of taking up the foot-pounds of work which are lost on account of the shock or impact which occurs when the shears begin to cut.

In calculating d under the suppositions (*a*) and (*b*) make use of the following results of *Dr. Hartig's* experiments on punching and shearing machines. A single acting machine when running empty will require the power,

$$H_e = 0.1 + \frac{nt^2}{26.7}, \tag{309}$$

and when at work will require, in addition to energy required to drive the tool when empty,

$$H_c = \frac{AF_w}{33000 + 50} = \frac{AF_w}{1980000}. \tag{310}$$

$H = H_e + H_c =$ total number of horse-powers.
t = maximum thickness of plate to be cut (in inches).
n = number of cuts per minute.

A = area of surface cut or punched per hour □″.
F_w = $(1160 + 1691t)$ foot pounds = a factor expressing the work required to produce a cut or sheared surface of 1 □″

Fig. 11.—*Horizontal and Conical Pivots.*

By allowing the pivot to enter but a short distance into its bearing the work of friction will be less than if cylindrical journal were used. This form of pivot is always employed for lathe centers because it is so well adapted for maintaining the line of centers; when used for this purpose the angle at apex = 60°.

It is also used on swiftly running arbors subjected to very light loads, as with emery wheels; with these wheels, however, it is much better to employ cylindrical journals, on account of protection from dust and emery afforded by the latter. This protection will much more than balance the theoretically greater friction of the cylindrical journal.

Assume $P = 30$ pounds, $n = 2500$. Angle at apex = 60°.

Fig. 12.—*Vertical and Conical Pivot.*

Here the work of friction can be made less than that arising with the flat pivot, provided the conical pivot enters but a short distance into its bearing. This form is much used for the pivots of spinning machines.

Assume $P = 20$ pounds, $n = 4000$. Angle at apex = 90°.

Fig. 13.—*Spherical Segment, or Cup and Ball Pivot.*
(Cast Iron or Lignum Vitae.)

By making the pivot slightly more convex than its bearings the work of friction will be reduced, as in the case of conical pivots; but after wear has fitted the rubbing surfaces to each other the friction will be greater than with the flat pivot. This form of pivot is principally useful when the vertical shaft which it supports is subject above to a slight lateral play, the pivot will

then act as a ball and socket joint. Assume same value for p as for flat pivot on Lignum Vitæ.

P = weight of turbine = 2240 pounds; n = 300. Angle at center = 90°.

Fig. 14.—*Flat Pivot, with Disks for Reducing Speeds.*

If ν = number of disks employed, then will $\frac{n}{1+\nu}$ represent the relaative speed of the two surfaces in contact.

Suppose the weight and load on a light vertical arbor making 5000 revolutions per minute to be equal to 50 pounds, and that it is desirable to make the diameter d = or $<$ 1 inch.

Find the number of speed disks which will keep wear within proper limits. It would be well to make the disks either of gun metal or of phosphor bronze.

Diameter of oil-hole = $\frac{5}{16}''$, $h = \frac{1}{6}d$.

Fig. 15.—*Ordinary Flat Pivot on Gun Metal.*

Diameter of oil reservoir = $\frac{1}{3}d$. Load on vertical shaft = 1 ton.
Diameter of oil channel = $\frac{1}{12}d$. $h = \frac{1}{6}d$, n = 150.

Determination of the Diameters of Shafts by the Help of Graphical Statics.

By the methods of Graphical Statics the bending or twisting moment to which any section is subjected can be determined and substituted in the following formulas:

When shaft is subjected to bending stresses only, we have

$$d = 2.167 \sqrt[3]{\frac{M_b}{f_b}}. \qquad (311)$$

When the forces which produce bending (all the forces being supposed to be projected on a plane at right angles to the axis of the shaft) make angles with each other, we can combine them—two at a time—by means of the formula,

$$M_b = \sqrt{M_{1b}^2 + M_{2b}^2 + 2M_{1b}M_{2b}\cos\gamma}, \qquad (312)$$

γ being the angle included between the forces whose bending moments are M_{1b} and M_{2b}.

When shaft is subjected to twisting stresses only,

$$d = 1.72\sqrt[3]{\frac{M_t}{f_t}}, \qquad (313)$$

M_t being the twisting moment to which the piece is subjected.

When shaft is subjected to combine bending and twisting we can use either the first or the third of the formulas just given—(307) or (309)—provided we substitute for M_b and M_t respectively, the equivalent or ideal bending moments $(M_b)_i$ and $(M_t)_i$, the values of which can be obtained by either calculating or constructing graphically the following formulas:

$$(M_b)_i = \tfrac{3}{8} M_b + \sqrt{(\tfrac{5}{8}M_b)^2 + (\tfrac{5}{8}M_t)^2}, \qquad (314)$$

$$(M_t)_i = \tfrac{5}{8}M_b = \sqrt{(M_b)^2 + (M_t)^2}, \qquad (315)$$

We can use either $d = 2.167\sqrt[3]{\dfrac{(M_b)_i}{f_b}}, \qquad (316)$

or $d = 1.72\sqrt[3]{\dfrac{(M_t)_i}{f_t}}. \qquad (317)$

When $(M_b)_i$ and $(M_t)_i$ are to be calculated for any section of the shaft we can use the approximations,

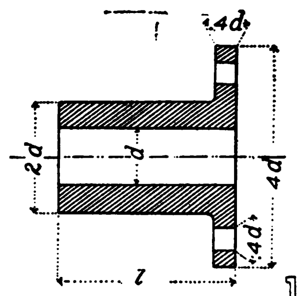
$$\text{When } M_b > M_t \begin{cases} (M_b)_i = 0.975M_b + .25M_t, & (318) \\ (M_t)_i = 1.560M_b + .40M_t. & (319) \end{cases}$$

$$\text{When } M_b < M_t \begin{cases} (M_b)_i = 0.625M_b + .60M_t. & (320) \\ (M_t)_i = M_b + .96M_t. & (321) \end{cases}$$

When $M_b = PA$ and $M_t = PR$.

$$\text{When } M_b > M_t \begin{cases} (M_b)_i = (.0975A + .25R)P, & (322) \\ (M_t)_i = (1.56\ A + 0.4R)P, & (323) \end{cases}$$

$$\text{When } M_b > M_t \begin{cases} (M_b)_i = (0.625A + 0.6R)P, & (324) \\ (M_t)_i = (A + .96R)P, & (325) \end{cases}$$



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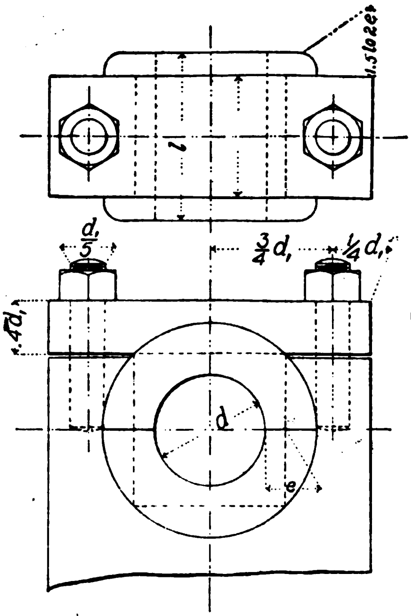
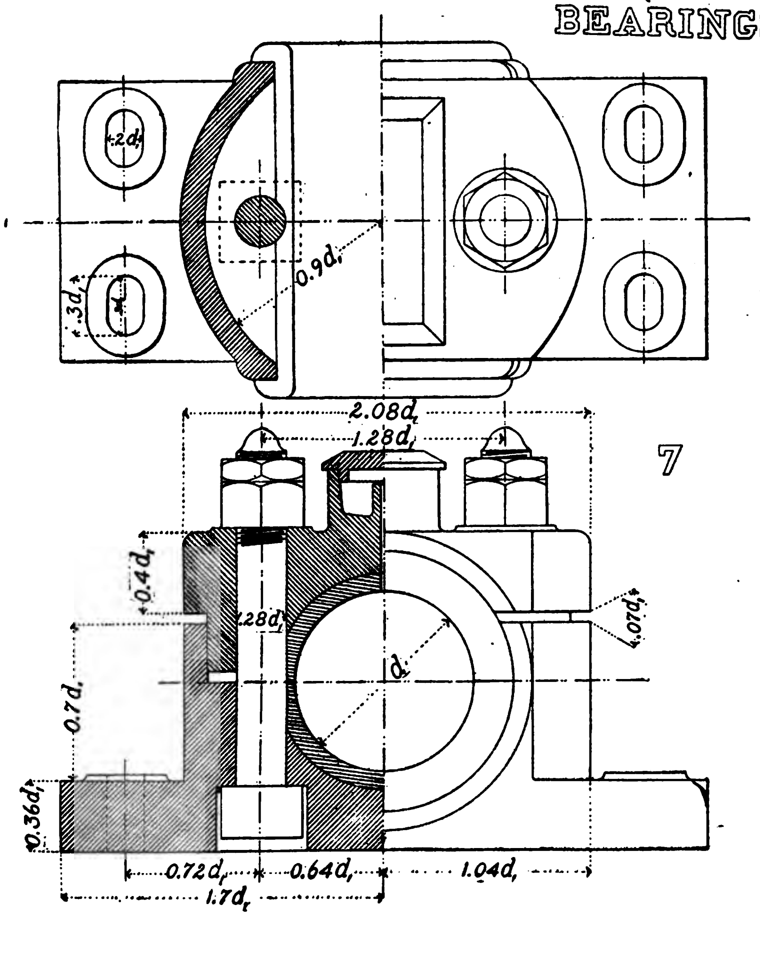


2

BEARINGS FOR RC

XIV



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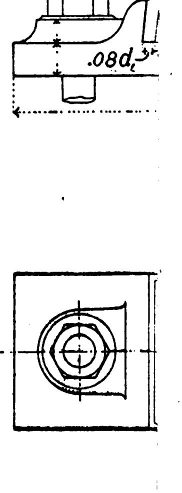


PLATE XIV.

BEARINGS FOR JOURNALS AND SHAFTING.

The supports for carrying journals and shafts are called *pillow* or *plummer blocks, pedestals, wall boxes* and *hangers,* while the part on which the journals run and rub are called *bearings,* although that name is often extended to the whole support.

Bearings for shafting and shafts should always be provided with two kinds of adjustments, namely, those for taking up wear and those for making the axes of shaft and bearing coincide. The wear must be taken up, otherwise the play between journal and bearing will cause inaccuracies of motion due to the shiftings of the axis of the shaft, and when the speed is very great the shocks and vibrations which ensue will be accompanied by loss of energy and rapid deterioration of the working parts. The means usually employed for taking up the wear, consists in dividing the bearing into two or more parts and then bringing the parts into closer contact by means of bolts or keys. It is principally in the cheapest kinds of work that bearings are not divided, and then only when the forces are small and speeds low; the other exceptions are the joint pin or small link-work bearings, which have only a slow oscillating motion, and the bearings of regulating and disengaging gear, which, from their being used intermittently, are subject to but little wear.

The adjustment for bringing the axis of the bearing exactly into line with that of its shaft is also very important. In the case of shafting this can be easily shown, for if we imagine the centers of these bearings *A, B, C,* out of line, the shaft at *B* will be exposed to a reaction of the support which will be proportional, or equal, to the force necessary to produce the deflection

of B from the straight line AC; if this is repeated at every three bearings, a long line of shafting will consume a great quantity of power and will heat and rapidly destroy the bearings. In like manner in engine shafts, if the running shaft does not rest truly all along its support, it will heat, for any irregularity in the relative positions of bearing and shaft causes the latter to rest upon a corner or edge of the brasses, where it exerts so great a pressure as to drive out the lubricant and causes an exceedingly rapid wearing away. The general direction in which adjustments for wear should take place is of course in the direction of the pressure, for it is evident that wear at or in the dividing plane cannot be taken up by bringing the brasses nearer together.

The amount of wear on journal and bearings depends partly upon the materials of which the two rubbing surfaces are composed, the pressure per $\square''$ against these surfaces, the speed of the rubbing and the efficiency of the lubrication. The material of the bearing is to be chosen principally with reference to diminishing the friction at the surface of the journal, not so much that the loss of mechanical work may be smaller, but to reduce the wear as much as possible. For this purpose a soft material, which but slightly attacks the journal, should be chosen, for, although its own wear may be considerable, it will prevent wear on the journal which (since it is a part of a more extended rotating piece) it would usually be more troublesome and costly to renew. The so-called "brasses" are therefore so constructed as to allow an exchange, of *worn* out for *new* ones, to be readily made. Brass is not employed, it is too hard and unyielding, but Bronze or Gun Metal, White Brass, Babbitt Metal and Phosphor Bronze are employed. Cast Iron bearings are used for the cheapest kinds of work, but are not to be recommended for general use, for although the coefficient of friction between wrought iron and cast iron is small, the latter attacks and wears out the journal quite rapidly.

Mr. Sellers maintains that it makes very little difference what material is employed for bearings, provided the bearings are long (that is, the pressure per □″ of rubbing surface is small) and the lubrication continuous, for then the shaft or journal rests on a film of oil and the materials do not come into contact. This view is spreading rapidly among engineers at the present time; the tendency is to make the lubrication so generous that only the internal friction of the particles of the lubricant will constitute the resistance. We will here give a table (prepared in part by *Dr. C. Künzel*) of the wear of materials used as bearings in axle boxes,

Kind of Alloy Employed for Bearing.	Distance (in *kilometers*) traversed by car to which axle boxes were attached, while *one kilogram* of metal was worn off in 4 axle boxes.	Wear on bearings of 4 axle boxes, for every 1000 kilometers of travel of car (in grammes).
Gun Metal, 83 copper, 17 tin,	90390	11.06
Gun Metal, 82 copper, 18 tin,	99900	10.01
White Metal, 3 copper, 90 tin, 7 antimony,	72280	13.83
White Metal, 5 copper, 85 tin, 10 antimony,	88145	11.34
Lead Composition, 84 lead, 16 antimony,	81280	12.30
Phosphor Bronze,	429200	1.33
Gun Metal on brake cars, 82 copper, 18 tin,	9134	109.48
Phosphor Bronze on brake cars,	107410	9.31
Parson's White Brass,	385275	2.60
Dewrance's Babbitt Metal,	637679	15.7

Reuleaux' "Konstructeur," p. 249, 4th Ger. ed., gives a valuable experiment on the wear of steel journals and Gun Metal bearings.

Composition of Colt Co.'s Babbitt Metal: Tin, 10 pounds; copper, 1 pound; antimony, 6 ounces. The copper is melted first, antimony added next and the melted mixture stirred, after

which the tin is added and whole mixture well stirred. It is from this material that the solid babbitt boxes, used as bearings for the crank pins and cross-head pins, were cast. The composition may be expressed in terms of 100 parts alloy, thus: Tin, 88 parts; Copper, 8.8 part; Antimony, 3.2 parts.

The article by *Dr. Künzel* (Polytechnisches Centralblatt, 1875; also "Engineer," November 26, 1875) from which most of the table on the preceding page was obtained, contains also the following remarks on bearings:

"Two bodies rubbed on one another with the same pressure and velocity, are heated more the greater their hardness. If the two are of different hardness the production of heat is less the greater the difference between their hardnesses; and for equal volumes the less hard body is heated less than the harder. Thus if glass be rubbed against cork the heating of the two bodies will be in proportion of 7 to 1. Hence, if we had not to take account of heating and wearing out of the axle, it would suffice, for realizing the ideal of a desirable bearing, to make it of the same metal as that of the axle. On the other hand, if we could neglect wearing out of the bearing and its deformation through pressure, a bearing made of very soft metal, in which turned an axle of the hardest kind possible, would realize the ideal of a bearing which would be little heated and would not attack the axle. In practice the point is to find a just medium between these extremes and to employ only bearings (1) which do not attack the axle; (2) which are themselves very little liable to wearing away, and consequently require very little greasing; (3) which are heated the least possible even where greasing may have been neglected; (4) which resist pressures and shocks to which they are exposed without changing form or breaking."

"The consumption of oil is in direct proportion to the wearing of the axle on the bearings."

Sometimes hard woods, and particularly lignum vitæ, have been employed for bearings. Lignum vitæ, has often been substituted for metal bearings when the latter gave trouble by heat-

ing, and not only when rotating pieces were employed but also for the guides of sliding pieces.

In the formulas for journals and pivots the permissible pressure per square inch of projected bearing surface was taken into account, also the speed or number of revolutions of the journals and shafts, but it will be well to call attention to the manner in which the speed of the rubbing surfaces may also affect the lubrication. The contact between a shaft and its bearing differs from sliding contact, as represented by the motion of a crosshead on its guides, in the following particular; in the latter case the action has a direct tendency to expel the lubricant and if the pressure is greater than the film will bear, the lubricant will be driven out, contact will take place between the metals, and rapid wear will ensue; but when a cylindrical shaft rotates on a similar bearing, the oil is driven up on each side and is repeatedly carried over by the retreating side of the shaft and reintroduced between the rolling surfaces as a film upon the advancing side. If the shaft under consideration is driven at a high speed centrifugal force comes into play and the film of oil acquires a pressure which will cause it to escape in the direction of the least resistance. The influence of the centrifugal force is particularly felt in the case of the crank pins of high speed engines, for there the velocity due to the crank's rotation is much greater than the relative velocity of rubbing between crank pin and connecting rod.

Fig. 1.—*Bushed Cast Iron Bearing.*

A bush is a sleeve or hollow socket which is adapted to receive a spindle, journal or pivot. It is usually placed in a hole cast in the frame of the machine. Since this device has no means for taking up the wear it must be replaced by a new bush when worn out. When made of cast iron it is used principally in the cheaper kinds of work. Bushing is a lining of gun metal which is used as a bearing in light link work, and is not divided after the manner of ordinary bearings. It is sometimes called a bush. The bolt holes shown in the figure are for fasten-

ing the bush to the frame of the machine. Means for lubricating the journal should also be provided—a hole drilled through the sleeve will answer. d and l must be calculated as they would be for the journal which is to run in the bearing.

Assume $P = 250$, $n = 120$.

If l should be small make it equal to 2″, and call the new length, l', then find the diameter d' which will give the same strength as the journal having the dimensions d and l; this may be done by means of the formula,

$$\frac{l'}{l} = \left(\frac{d'}{d}\right)^3. \qquad (326)$$

Fig. 2.—*Square Box of Babbitt Metal.*

Here the wear on the boxes can be taken up by screwing down the cap or cover of the box. In this case the babbitt is not simply a lining, but forms the box itself. Such an arrangement would not be suitable if the journal were subjected to or carried great loads. Calculate as for a wrought iron journal running on gun metal bearings. The wear of good babbitt metal is probably considerably smaller than gun metal. The unit is no longer d as in the first figure, but is

$$d_1 = 1.15d + 0.4,$$

this unit being employed for the block or pedestal, while another,

$$e = 0.09d + \tfrac{1}{8}'',$$

is the unit to which the dimensions of the brasses or boxes are referred. Assume $P = 1$ ton, $n = 180$.

Also draw some device for lubricating the journal continously.

Fig. 3.—*Ordinary Cast Iron Bearing, Forming Part of Machine's Frame.*

This kind of bearing is still much used in cheap work; it is better practice to babbitt them by pouring babbitt metal (around a shaft the same size as the journal) into recesses left for the

purpose. When the pressures are large it is customary to recess only a portion of the cast iron bearing, so that the journal will rest partly on cast iron and partly on babbitt. In such cases the recesses should be helical, and not simply grooves at right angles to the axis of the shaft, for in the latter case the journal might be unequally worn, having shallow grooves where it came in contact with the cast iron. Draw this particular bearing with a recess for babbitting extending nearly the whole length of the journal and having a depth of $\frac{3}{32}''$.

Assume $P = 2500$, $n = 75$, $\frac{l}{d} = 1.5$, $d = 0.04\sqrt{P}$.

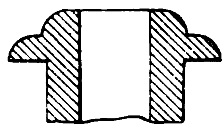
Unit to be employed $= d_1 = 1.15d + 0.4$.

Figs. 4, 5 and 6.—*Brasses.*

They are made of gun metal and will therefore wear more rapidly than the journal which is of harder material; when worn out they can be replaced much more cheaply than the journal and the rotating piece to which the latter is attached. Brasses must be so arranged that they will not turn with the shaft or journal, those portions, therefore, which come in contact with the body of the pedestal or plummer block are made octagonal or square. They are sometimes wholly cylindrical and are prevented from turning by bolts passing through them or by projecting pins. See *Unwin,* p. 248, and *Reuleaux,* pp. 281-282. Brasses should also be liberally supplied with oil grooves, so that oil may be conducted to every part of the journal. One oil groove should always be put parallel to the axis in the upper half of the brass, where it will be in direct communication with the oil holes; then the advancing side of the journal will always carry a film of oil between the rubbing surfaces. The oil grooves should not be less than $\frac{1}{8}''$ even in small bearings and may increase to $\frac{5}{16}$ in large ones; this is particularly necessary with soft metal bearings, as the rapid wear of the latter tends to clog up the oil grooves and holes.

The brasses are first cast in halves, then lightly soldered at the dividing plane, and, while thus united, turned.

The proportions given in the figure will answer for $\frac{l}{d} = <$ or 2. *Reuleaux* employs the dimensions $0.9d_1$ where $0.72l$ is employed in Fig. 4, giving to the flange externally an *OG* profile:



Assume $d = 3\frac{1}{2}''$, $l = 6''$.

Fig. 7.—*Compact Pillow Block.*

The pedestal was designed by *Mr. Arthur Rigg, C. E.* Its proportions are given by *Unwin*, p. 250, the units being

$$d_1 = d + \tfrac{1}{2}, \qquad \text{and} \qquad e = 0.1d + \tfrac{1}{8}''.$$

Almost all the work on the block, as well as on the brasses, can be done on the lathe, *i. e.*, can be either turned or bored; this enables the pillow block to be accurately and cheaply made. The brasses can at once be snugly fitted to the body without the intervention of hand filing, which is usually the case when part of the work has to be planed as in Fig. 4.

The brasses are kept from turning with the shaft by bolts passing through them. The proportions given in the figure on the plate will give a figure like that shown in *Unwin*, p. 250, when the relations between d and l are as shown in the following table:

$d =$	3	4	5	6	7	8
$e =$	0.43	0.53	0.63	0.73	0.83	0.93
$d_1 =$	3.5	4.5	5.5	6.5	7.5	8.5
$l =$	5.84	7.38	8.92	10.48	12.02	13.26
$\frac{l}{d} =$	1.95	1.85	1.78	1.75	1.72	1.70

When l is greater than given in the table the flange of the brass may be given a greater thickness, for example, that adopted by *Reuleaux*, *i. e.*, $\frac{l - 0.9d_1}{2}$ the profile of the flange being an *OG* curve; or the arrangement shown in the plate may be adopted. If we are willing to sacrifice compactness we can increase the radius to which the cylindrical recess of the body for receiving the cap is turned, so long as uncomely proportions are not obtained. It corresponds with good practice to insert between cap and pedestal a thin strip of gun metal just fitting the fissure between them which was left for the purpose of taking up the wear. When the wear of the brasses has become so large that it must be taken up, the strips are taken out and made thinner by filing down an amount equal to wear. When replaced the cap can be screwed down without danger of gripping the shaft too tightly.

Draw the body of the pedestal—wherever it is concealed by the brasses or the cap—in dotted lines.

Assume $P = 10000$. $n = 250$. Wrought iron on gun metal.

Fig. 8.—*Pedestal with Foundation Plate.*

Unit $= d + \frac{1}{2}''$.
Assume $d = 6''$.

Fig. 9.—*Brasses for Fig.* 8.

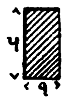
CONNECTING RODS.

b = breadth of rectangular rod.

b = also breadth of rod in I section, in this case $b = cH$.

b_1 = also breadth of web of I rod $= c'H$.

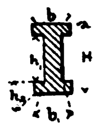
d = diameter of crank pin.



D = diameter of circular rod at middle section.

f = allowable working stress in pounds per □″.

h = height of rectangular rod at middle section.



H = total height or depth of rod with I section.

h_1 = height of web of I rod $= c''H$.

h_2 = height or thickness of flange of I rod $= c'''H$.

$$h_1 + 2h_2 = H. \quad (327)$$

$\frac{J}{a}$ = modulus of cross-section for I rod

$$\frac{J}{a} = \frac{bH^3 - (b - b_1)h^3_1}{6H} \quad (328)$$

l = length of connecting rod between center of crank pin and center of cross-head pin.

P = maximum load acting in direction of rod.

q = area of cross section at middle of rod

$$q = bH - (b - b_1)h_1 \quad (329)$$

r = length of crank in inches.

V = average piston speed in feet per minute.

Z = factor for circular section obtained by assuming average stresses and dimensions.

Z' = ditto for rectangular section.

Z'' = ditto for I section.

$$X = \text{numerical factor} = \frac{H^2}{q} = \frac{H^2}{bH - (b-b_1)h_1}$$

$$= \frac{H^2}{[e - (c - c')c''] H^2} = \frac{1}{c - (c - c')c''}.$$

$$y = \text{numerical factor} = H^3 \div \frac{J}{a} = \frac{6}{c - (c - c')(c'')^3}.$$

Determination of Cross-section of Connecting Rod.

(Taken from an article by *Josef Bartl*, "*Der Civil Engeniur*," 1879).

The formulas hitherto given (*Reuleaux*' for example) for determining the cross-section of connecting rods do not contain the allowable working stress per □″, this indetermination is covered usually by a larger factor of safety. To remove this indetermination and empiricism *Mr. Bartl* has made a rigorous investigation of all the stresses to which the rod is subjected, taking also into account the effect of wear in brasses at crank and cross-head pins.

Mr. Bartl first shows that the position of the dangerous cross-section in all kinds of connecting rods varies but little, the variation of its distance from the crank pin being confined between the limits $0.36l$ and $0.43l$. He therefore assumes the distance $0.4l$ from the crank pin center, as the location of the dangerous cross-section. In order to still further reduce the complicated formulas arrived at, average values were employed and the following proportions:

For Single Crank

$$\left.\begin{array}{l}\text{Diameter of iron crank pin} = d = .03070\sqrt{P} \\ \text{Diameter of steel crank pin} = d = .02181\sqrt{P}\end{array}\right\}$$

$$\frac{\text{diameter of cross-head pin}}{\text{diameter of crank pin}} = 0.75 \qquad (336)$$

For Double Crank.

$$\left.\begin{array}{l}\text{Diameter of iron crank pin} = .06141\sqrt{P} \\ \text{Diameter of steel crank pin} = .04361\sqrt{P}\end{array}\right\}$$

$$\frac{\text{diameter of cross-head pin}}{\text{diameter of crank pin}} = 0.4. \qquad (331)$$

The expressions for Z, Z' and Z'' are of such form that the values obtained by substituting *average* stresses and dimensions for the *actual* ones, deviate but slightly from the exact results.

When Cross-Section of Rod is Constant.

$$D = Z\sqrt{\frac{P}{f - 1.109\frac{l^2}{D}\left(.0000363\frac{V^2}{r} + .268\right)}}.$$

$$h = Z'\sqrt{\frac{P}{f - .0832\frac{l^2}{h}\left(.0000363\frac{V^2}{r} + .268\right)}}.$$

$$H = Z''\sqrt{\frac{PX}{f - 0.138\frac{\gamma l^2}{XH}\left(.0000363V^2 + .268\right)}}.$$

Single Crank.

Cross Section of rod.	Values of Z		Values of Z'.										Values of Z''. Steel.
			Iron.					Steel.					
	Iron	Steel	$\frac{h}{b}=1.6$	1.8	2.0	2.2	2.4	$\frac{h}{b}=1.6$	1.8	2.0	2.2	2.4	
●	1.17	1.20											
▮			1.29	1.37	1.44	1.51	1.58	1.32	1.39	1.40	1.54	1.90	
I													1.02

DOUBLE CRANKS.

Cross Section of rod.	Values of Z.		Values of Z'.										Values of Z''. Steel.
			Iron.					Steel.					
	Iron	Steel	$\frac{h}{b}=1.6$	1.8	2.0	2.2	2.4	$\frac{h}{b}=1.6$	1.8	2.0	2.2	2.4	
●	1.25	1.34											
▮			1.36	1.43	1.51	1.57	1.64	1.43	1.50	1.58	1.65	1.72	
I													1.07

Inspection of the three formulas just given, shows that the quantities D, h and H sought, exist in the second members of the equations. In order to make use of the formulas we assume each of the ratios $\frac{l}{D}$, $\frac{l}{h}$ and $\frac{l}{H}$ equal to the average value 20, then calculate D, h or H. This will give a new value for $\frac{l}{D}$ or $\frac{l}{h}$ or $\frac{l}{H}$. Substitute this new value and again calculate D, h or H, and so on until a sufficiently exact result has been obtained.

It is customary in practice to take a larger factor of safety for small engines than for larger ones; it is also customary to take this factor larger for stationary than for locomotive engines.

The allowable working stress per □″ $(=f)$ may be taken as follows:

For *wrought iron* connecting rods and
small and average sized stationary engines $f = 1400$ to 2100.
For large sized stationary engines $f = 2100$ to 2800.
For locomotive engines $f = 2800$ to 3600.
For *steel* connecting rods and
For stationary engines $f = 4300$ to 5700.
For locomotive engines $f = 6400$ to 7300.

The values of f are, to be sure, considerably smaller than those employed for most of the other machine parts, but we are

justified in assuming them because of the extremely rapid and sudden changes of stress to which the connecting rod is subjected during every revolution of the crank. The formulas previously given for the values of D, H and h, were obtained on the supposition that the cross-section of the rod was uniform, but calculation shows that they may also be employed for rods of varying cross-section under the following circumstances:

When the rod is of circular cross-section the middle section must be given a diameter D calculated from the first of the above formulas, the diameter of section at or near crank pin must be $0.8D$. There is a uniform taper from middle to each end of rod. The maximum stress at the dangerous (middle) section will be slightly smaller than if the rod were uniform throughout.

When the rod is of rectangular section the breadth b must be constant, while the height or depth h varies uniformly from $\frac{4}{5}h$, at cross-head pin, to $\frac{6}{5}h$, at crank pin. When tnis is the case and $\frac{h}{b}$ differs but little from $\frac{1}{0.6} = \frac{5}{3}$, the maximum stress at dangerous cross-section differs but little from that for constant cross-section, consequently h for middle section is given by second formula.

Marine engine rods are usually of short length and have very large ends; consequently the above formulas are not applicable. In such cases we can employ the formulas given by *Reuleaux*, and introduce large factors of safety (from 30 to 80 according to *Reuleaux*. Otherwise the laborious task must be undertaken of ascertaining by graphical means the influence of the large end masses on the maximum stress to which the rod is subjected.

INDEX.

APPENDIX.

GEAR TABLES.

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Example (See Fig. 1) 5 2 4 .300 .065 Lay as in

TABLE I.

NUMBER OF EQUIDIS 9 12 13

Distances from horizo 1.35 1.80 95

Distances from vert 9 .4598 .72830 294

TABLE II.

NUMBER OF EQUIDIST 5 6 9 18

Distances from horizon .360 .500

Distances from vertica 30 .110 .18622

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TABLE III.

dendum equals $\frac{1}{\cdot}$ diam. pitch.

NUMBER OF EQUIDIS 7 5 10

Distances from horizonta .178500

Distances from vertical .0529058

FIG. 1. (See example, table I.)

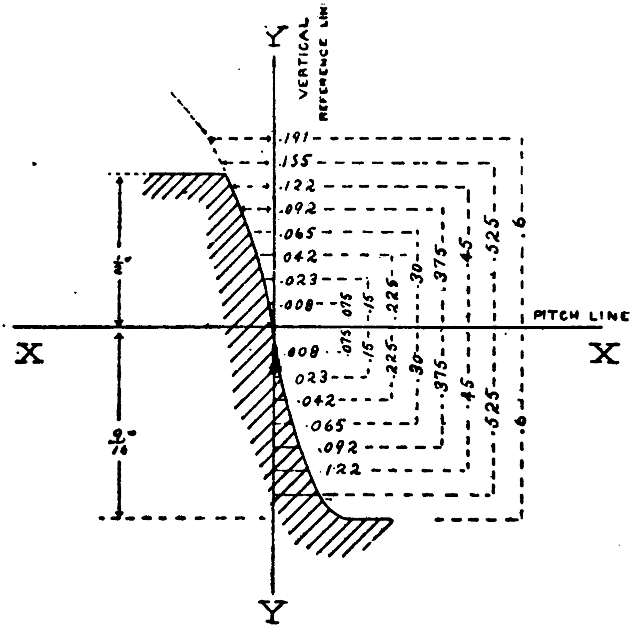

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TABLE VII.

SUPPLEMENTARY INVOLUTE TABLE, TO BE USED WITH TABLE V, FOR LOCATING REFERENCE LINES; ALSO FOR LAYING OUT THE STRAIGHT PROFILE OF ANY INVOLUTE RACK.

$$m = b_1 M_1 - a_1 \sqrt{0.25 - M_1^2},$$

$$n = \frac{1}{2}\left[\frac{1}{q} - q\right] \quad \text{when} \quad q = 2a_1 M_1 + 2b_1 \sqrt{0.25 - M_1^2}$$

Value M_1 is obtained from Table VI, for given backlash and number of teeth.

m and n multiplied by diameter of base circle, give location of the reference line which is tangent to base circle, m corresponding to distance CO and n to distance CD of Fig. 4.

Angle made by Line of Thrust with Line of Centers.	Diameter of Base Circle Divided by Diameter of Pitch Circle.	Factors in above Formulas. Factor a_1.	Factors in above Formulas. Factor b_1.	*Involute Racks.* Distances from Vertical Reference Line Divided by Distances from Horizontal Reference Line.
70°	.93969	.01491	.99989	.3640
71°	.94552	.01272	.99992	.3443
72°	.95106	.01076	.99994	.3249
73°	.95631	.00902	.99996	.3057
74°	.96126	.00749	.99997	.2867
75°	.96593	.00615	.99998	.2679
76°	.97030	.00498	.99999	.2493
77°	.97437	.00398	.99999	.2309
78°	.97815	.00312	1.0000	.2126
79°	.98163	.00240	1.00000	.1944
80°	.98481	.00179	1.00000	.1763

Values in second column multiplied by diameter of pitch circle, give diameter of base circle.

Values in fifth column multiplied by length of tip of tooth, give distance of tip of straight profile from vertical reference line.